Multi-input downstream firms and vertical contracting

Simen Aardal Ulsaker*

VERY PRELIMINARY

May 13, 2014

1 Introduction

Downstream firms often have multiple sources of input. Retailers typically stock the products of multiple producers, some of which they may be vertically integrated with, and downstream manufacturers often use inputs provided by several upstream suppliers. The present article studies the interaction between two competing downstream firms and a common upstream manufacturer, while allowing the downstream firms to have several sources of input: If a downstream firm fails to reach an agreement with the common manufacturer, it is assumed still to be able to earn positive profits, through the use of other inputs than the one offered by this common manufacturer.

In the framework of this article, the profit a downstream firm can make if it does not trade with the common manufacturer depends on the trade between the rival downstream firm and this manufacturer. This will prove important. When the downstream firms offer contracts there may be no equilibrium in which both downstream firms trade with the common manufacturer, even when the downstream firms are allowed to offer menus of contracts to the common manufacturer. This is in contrast to what would be the case with single-input downstream firms. When the manufacturer is the one offering contracts, she can use actual or potential exclusion of one of the downstream firms to extract more profit from the downstream firms than her “contribution” to the

*Norwegian School of Economics, simen.ulsaker@nhh.no.

industry profit. In contrast to what is the case with single-input firms, simple two-part tariffs is not sufficient for the manufacturer to achieve her preferred outcome.

The present article builds on the framework introduced by Marx and Shaffer (2007), who study an industry were two downstream firms make simultaneous contract offers to a common upstream manufacturer. The authors examine the role of upfront payments: transfers made by the manufacturer to a downstream firm when a contract is signed. They find that the stronger downstream firm will use upfront payments to exclude the weaker downstream firm by offering an appropriate "three-part tariff" consisting of an upfront payment, a fixed fee that is paid by the downstream firm to the manufacturer if and only if trade actually occurs, and a per-unit wholesale price. The contract is constructed such that the stronger downstream firm will decide to trade with the manufacturer if and only if the rival does not trade. To secure the trade with the stronger downstream firm, the manufacturer rejects the offer from the weaker downstream firm, and exclusion occurs in every equilibria of the game.

Miklós-Thal et al. (2011) show that if the downstream firms are allowed to make the contract offers contingent on exclusivity, there exists non-exclusionary equilibria that give the downstream firms higher payoffs than the exclusionary equilibria of Marx and Shaffer (2007). Rey and Whinston (2013) show that it is not necessary to allow the downstream firms to make offers contingent on exclusivity to achieve non-exclusionary equilibria: As long as the downstream firms are allowed to offer a menu of contracts, a non-exclusionary equilibrium exist in which industry profit is maximized and each downstream firm earns its "contribution" to this profit. As we shall see, these results no longer hold when downstream firms have multiple sources of input, in which case there might not exist non-exclusionary equilibria, even if the downstream firms are allowed to offer menus of contracts, and even if these contracts can be made contingent on exclusivity.

The downstream firms will only choose to buy from the common manufacturer if this leaves them with a higher profit than the alternative, which is only using the other available inputs. Importantly, the value of this outside option depends on the contract between the rival downstream firm and the common manufacturer. The outside option if the rival has a contract intended for common agency, where the wholesale prices typically are above marginal cost in order to dampen downstream downstream competition, is generally more profitable than the outside option when the rival has offered a contract intended for an exclusive relationship with the manufacturer, in which case wholesale price is typically equal to marginal cost. With three-part tariffs, the flow profit of a given downstream firm in any equilibrium is bounded below by its outside option. This might make it profitable for the other downstream firm to induce an exclusive relationship with the manufacturer. Exclusionary equilibria will however always exist, and in certain situations, all equilibria are exclusionary.

When the market structure is such that only one downstream firm will trade with the common manufacturer, the downstream firms will in effect be bidding for exclusivity. In the equilibria of Marx and Shaffer, the weakest downstream firm offers the manufacturer its entire potential flow profit in a vain attempt to become the exclusive downstream firm. For multi-input downstream firms, the benefit of being the only firm trading with the common manufacturer is twofold. First, a downstream firm that is alone in dealing with the manufacturer will be the sole downstream beneficiary of the increased value the manufacturer's input brings to the industry. Second, the exclusive presence of the manufacturers's input at this firm might attract trade that otherwise would have taken place at the rival if both (or none) of the firms traded with the manufacturer. When the downstream firms are retailers, this could obviously be the case when product provided by the manufacturer is viewed as a substitute for some of the other products the downstream firms stock. It might also be the case when the manufacturers product is not viewed as a substitute for the other products sold by the downstream firms. If the consumers typically buy a basket of products from a given downstream firm, the fact that a given product is made available only at one firm might shift demand for a seemingly unrelated product towards this firm, at the loss of the rival downstream firm.

The argument above makes evident that the exclusive downstream firm's gain is partly the rival's loss: In an exclusionary equilibrium, the excluded downstream firm will be made worse off by the presence of the manufacturer. More surprising is perhaps that also the firm that *does* trade with the manufacturer might be hurt by her presence. A downstream firms knows that if it does not agree on a contract with the manufacturer, the rival probably will. The downstream firms will therefore bid intensively for exclusivity, not least to prevent the rival from obtaining the manufacturers product and capturing a larger share of the downstream trade. If the downstream firms are sufficiently symmetric, the bidding for exclusivity will be so intense as to leave both firms with a lower profit than they would have had if the manufacturer was not in the market.

The fact that it is costly for the downstream firms to let the rival end up in an exclusive relationship with the common manufacturer, can also be exploited by the manufacturer if she is the one who offers contracts to the downstream firms. If one downstream firm rejects the contract offer from the manufacturer while its rival accepts, it will generally end up with a lower profit than if neither of them traded with the common manufacturer. This will enable the manufacturer to extract more profit from the downstream firms than her "contribution" to the industry profit. The flip-side of this is that the downstream firms – jointly and individually – always are made worse off by the presence of the common manufacturer, when the manufacturer is the one who offers contracts.

In contrast to what is the case with single-input downstream firms, simple two-part, or even three-part tariffs, is not sufficient for the manufacturer to achieve the optimal outcome when she offers the downstream firms take-it-or-leave it contracts.[1] The complexity of the contracts available to the manufacturer affects not only the profits of the three firms, but also whether both downstream firms trade with the manufacturer, and at what marginal wholesale prices.

As mentioned above, the outside option of the downstream firms in a common agency equilibria with two-part tariffs or three-part tariffs can be quite substantial, since the wholesale prices typically are above marginal cost in order to dampen downstream competition. In fact, when the manufacturer is restricted to offering three-part tariffs, there often does not exists a common agency equilibrium at all. The fact that one downstream firm is made worse off if only the rival buys from the common manufacturer, can be exploited by the manufacturer when fixed fees that are contingent on trade can be included in the contracts. By, for example, offering one downstream firm a contract that is such that this firm will buy from the common manufacturer if and only if the rival rejects its contract offer, the manufacturer can make this latter downstream firm decide which firm ends up in a *de facto* exclusive relationship. By offering the appropriate contracts, this enables the manufacturer to "force" one downstream firm to accept a contract that in effect makes it the exclusive downstream firm, even when this firm is left only with a negligible profit. This strategy might enable the manufacturer to achieve a higher profit through exclusion than she is able to achieve through selling to both downstream firms, when restricted to offering three-part tariffs. The firm that is excluded is used by the manufacturer to degrade the outside option

[1]See Hart and Tirole (1990) for the case with two-part tariffs and single-input downstream firms.

of its rival, and the excluded downstream firm might well in equilibrium end up with a higher profit than the downstream firm that trades with the manufacturer.

A similar strategy of pitting the downstream firms against each other can however be used by the manufacturer to secure common agency equilibria, if she can offer the downstream firms contracts that lets the downstream firms choose between different two-part tariffs *after* they have chosen to accept or reject the contract offers. The manufacturer would then construct contracts that are such that one two-part tariff is chosen if the rival rejects its contract, and the other is chosen if the rival accepts its contract.

If the "surprise" in the model of Marx and Shaffer (2007) was "the helplessness of the manufacturer in preventing exclusion" (p. 830), the surprise in the framework of the present article is the inability of the downstream firms to gain from trading with the manufacturer. As discussed above, the downstream firms can in certain situations be made worse off by the presence of a manufacturer, even when they have all the bargaining power. When the manufacturer is the one who is offering contracts, the downstream firms will always prefer that she was absent from the industry altogether.

The article is organized as follows. In Section 2, the framework of the analysis is presented. Section 3 examines the situation where the downstream firms offer contracts. In Section 4 the common manufacturer is the one offering contracts. Section 5 discusses the effects of letting the downstream firms cooperate when interacting with the manufacturer. Section 6 concludes.

2 Framework

Two firms, R_A and R_B, have the opportunity to use the n products of the set $N = \{1, 2, ..., n\}$ as inputs when producing products they sell in a downstream market. The downstream firms do not incur any other costs in production than what they pay their suppliers for the inputs they choose to use. Let Product 1 be supplied by an upstream manufacturer called M. This firm incurs a cost $c(q_{1A}, q_{1B})$, where $q_{1i} \geq 0$ is the quantity sold of its product to R_i, $i \in \{A, B\}$. Let this cost function be non-negative and weakly increasing in each argument.

In what follows the emphasis is on the interaction between M and the two downstream firms. For simplicity, assume that Product 2 through n are available for R_i at a wholesale price that is not affected by the interaction between M and the two downstream firms. These products could for example be produced by competitive industries with constant marginal cost of production.

Following Marx and Shaffer, the downstream competition is modeled in reduced form. Let (w_A, w_B) be the unit prices at which the downstream firms can purchase Product 1. Assume that if both downstream firms purchase a strictly positive quantity from M, there exists an equilibrium in the downstream market and that the equilibrium flow profits are unique. Let $\pi_i(w_i, w_j)$ and $\pi_M(w_A, w_B)$ be the flow profit of R_i and M respectively, when R_A and R_B purchase Product 1 at prices w_A and w_B.[2] Let $q_{1i}(w_i, w_j)$ be R_i's equilibrium input demand of Product 1. It seams reasonable that if w_i is high enough, R_i will choose not to buy a positive quantity from M. Assume that $\pi_i(w_i, w_j)$ is decreasing in w_i and increasing in w_j, with these effects holding strictly when R_i and R_j, respectively, buy strictly positive quantities from M. This implies that for all (w_A, w_B) such that both downstream firms purchase from the manufacturer, $\pi_i(w_i, \infty) > \pi_i(w_i, w_j)$. Assume also that the marginal effect on own profit when the rival's wholesale price is increased (which is positive), decreases in own wholesale price:

Assumption 1. $\frac{\partial^2 \pi_i(w_i, w_j)}{\partial w_j \partial w_i} \leq 0$ for all (w_i, w_j). $\frac{\partial^2 \pi_i(w_i, w_j)}{\partial w_j \partial w_i} < 0$ for all (w_i, w_j) such that both firms buy from M.[3]

If M does not trade with it, R_i gets a flow profit of $\pi_i(\infty, w_j)$ from using the other available inputs. Assume this payoff is weakly positive, that is that $\pi_i(\infty, w_j) \geq 0$ for all w_j. If neither R_A nor R_B buys from M, R_i gets a profit of $\pi_i(\infty, \infty)$. Note that the assumptions above imply the following.

Lemma 1. $\pi_i(\infty, \infty) > \pi_i(\infty, w_j)$ for all w_j such that $q_{1j}(w_j, \infty) > 0$.

This lemma states that the rival's use of product 1 reduces R_i's profit even when product 1 is not used by R_i. This will prove crucial in the following analysis and merits some consideration.

If the downstream firms are retailers in the sense that they place one unit of a given input on their shelves for resale, Lemma 1 implies that R_i's profit is reduced when the rival stocks product 1 even when R_i itself does not stock this product. This seems reasonable if the consumers consider product 1 a substitute for some of the products that R_i stocks. It could however also be the case when product 1 is *not* considered

[2] j refers to R_i's rival. Note that the prices of the other $n-1$ inputs enter these and all the following profit functions as parameters.

[3] This assumption is satisfied for many standard oligopoly models. For a discussion, see Assumption 3 in McAfee and Schwartz (1994).

a substitute for any of the products stocked by R_i. To see this, note that consumers typically get some (positive or negative) utility from purchasing a set of products from a given retailer, in addition to the utility they get from consuming the same set of products. Often there will be some economies of scope in making purchases: Buying a single product, say product 1 at a given retailer, say retailer R_A, might cost the consumer a substantial amount of both time and effort. But if the consumer already buys other products at R_A, the additional time and effort of also buying product 1 at retailer R_A might well be small. Such economies of scope leads consumers to prefer one-stop-shopping, which can explain why the availability of product 1 at the rival reduces the profit of R_i, even when not being viewed as a substitute for the products sold by R_i.[4]

The downstream firms of the model need however not be retailers in the classical sense. The input provided by the upstream firms may for example be content that is made available through downstream platforms.[5] Platforms often charge consumers a fee for (possibly time-limited) access to the platform. Here, there could also be economics of scope for the consumers when it comes to accessing content: The access fee will often be paid only once, regardless of the amount of content consumed.[6] The willingness to pay for access to a given platform will generally be a function of the content available through both platforms. If M's content becomes available at R_j's platform, then the willingness to pay for access to the platform of R_i might decrease, even when M's content is not a direct substitute for any of the content available through R_i, as long as some of the content available through the different platforms are viewed as substitutes by the consumers, and there are some economies of scope in accessing content through the platforms.

Finally, the downstream firms may be manufacturers proper, i.e., firms that substantially transform inputs provided by M and other upstream firms to produce products they sell on the downstream market. If the input supplied by M is nonessential,

[4]In many retail markets, for example the grocery market, consumers typically visits a retailer to buy a basket of goods. The retailers can then be thought of as offering basket of goods at certain aggregate prices. If a product is made available only at one retailer, the demand for the baskets available at other retailer could well decrease, even when product under consideration is not viewed as a substitutes for any of the other products in any of the baskets under consideration.

[5]Examples of such platforms could be television networks and networks distributors, streaming services, game consoles and operative systems (possibly including bundled hardware).

[6]In addition, to the disutility of paying this access fee, the consumer might get some utility from simply having access to a given platform. This utility might include utility of consuming some basic content that comes bundled with the platform access.

but reduces the marginal cost of production or increases the quality of the products manufactured by the downstream firms, then Lemma 1 seems reasonable.

The joint profit of M and the downstream firms is $\Pi(w_A, w_B) \equiv \pi_M(w_A, w_B) + \sum_i \pi_i(w_i, w_j)$. Let w_A^* and w_B^* be prices that uniquely maximize this profit, and denote the corresponding joint profit by Π^*.[7] Slightly abusing notation, denote the joint flow profit of and M and R_i by $\Pi_i(w_i, w_j) \equiv \pi_M(w_i, w_j) + \pi_i(w_i, w_j)$. Let $W_i(w_j) \equiv \arg\max_{w_i} \{\pi_M(w_i, w_j) + \pi_i(w_i, w_j)\}$. If only R_i buys from M, the joint flow profit of these two firms is $\Pi_i(w_i, \infty)$. Let w_i^m be the wholesale price that maximizes this profit, and write the maximized joint profit as Π_i^m. Note that $w_i^m = W_i(\infty)$. It should be clear that $W_i(w_j) \geq w_i^m$ for all w_j.

Finally, assume that downstream firms are imperfect substitutes in generating industry profit, that is that:

Assumption 2. $(\Pi_A^m + \pi_B(\infty, w_A^m)) + (\Pi_B^m + \pi_A(\infty, w_B^m)) > \Pi^* > \max\{\Pi_A^m + \pi_B(\infty, w_A^m), \Pi_B^m + \pi_A(\infty, w_B^m)\}$.

For this to hold there has to be some substitutability in upstream manufacturing costs or in downstream demand.

3 Downstream firms offer contracts

This section considers properties of equilibria in a game where the downstream firms simultaneously offer the common manufacturer contracts. Following Marx and Shaffer (2007) contracts are assumed to be "three-part tariffs".

Consider the following game.

(i) R_A and R_B simultaneously make contract offers to M.

(ii) M accepts or rejects the downstream firms's offers. Accepted contracts are public information.

(iii) Downstream firms with accepted contract offers decide how much to buy from M. Both downstream firms decide how much to buy of the other available inputs. Production and downstream competition takes place, and finally payments are made according to accepted contracts.

[7]Note that this industry profit is only maximized with respect to the wholesale price of product 1. The wholesale prices of the other inputs will not necessarily be at the industry profit maximizing levels.

Assume that the contracts offered to M consists of three-part tariffs $T_i(q_{1i})$ of the following form:

$$T_i(q_{1i}) = \begin{cases} S_i & \text{if } q_{1i} = 0 \\ w_i q_{1i} + F_i + S_i & \text{if } q_{1i} > 0. \end{cases}$$

S_i is a fixed, possibly negative, payment paid from R_i to M. The component $w_i q_{1i} + F_i$ is a two-part tariff paid to M if and only if R_i decides to buy a strictly positive quantity from the manufacturer. Denote such three-part tariffs by $T_i = (S_i, w_i, F_i)$. Following Miklós-Thal et al. (2011) it is assumed that a given contract can be contingent on exclusivity. Take this to mean that the contract includes an "exclusive territory" clause stating that the downstream firm is only obliged to pay M according to the contract if the rival does not buy from M in stage (iii). Assume that in stage (i), it is possible for the downstream firms to offer the common manufacturer a menu of contracts to choose from in stage (ii). Some or all of these contracts may be contingent on exclusivity.

Let us consider stage (iii) of the game, and solve for equilibrium outcomes in this stage. In this stage, downstream firms with accepted contracts must decide how much to buy from M. If and only if R_i decides to buy a strictly positive quantity from M will it have to pay F_i. There are two scenarios to consider. If M has accepted only a contract from R_i, this firm will buy a positive quantity (and pay the fixed fee) if and only if $\pi_i(w_i, \infty) - \pi_i(\infty, \infty) \geq F_i$. The second scenario to consider is the case where M accepts both offers.

If M has accepted contracts from both downstream firms, we have the following. Given that neither of the accepted contracts are contingent on exclusivity, R_i purchases from M if $\pi_i(w_i, w_j) - \pi_i(\infty, w_j) \geq F_i$ but does not purchase if $\pi_i(w_i, \infty) - \pi_i(\infty, \infty) < F_i$. If $\pi_i(w_i, w_j) - \pi_i(\infty, w_j) < F_i \leq \pi_i(w_i, \infty) - \pi_i(\infty, \infty)$,[8] then, in third stage equilibrium, R_i will purchase if $\pi_j(w_j, \infty) - \pi_j(\infty, \infty) < F_j$ but not if $\pi_j(w_j, w_i) - \pi_j(\infty, w_i) \geq F_j$. If $\pi_A(w_A, w_B) - \pi_A(\infty, w_B) < F_A \leq \pi_A(w_A, \infty) - \pi_A(\infty, \infty)$ *and* $\pi_B(w_B, w_A) - \pi_B(\infty, w_A) < F_B \leq \pi_B(w_B, \infty) - \pi_B(\infty, \infty)$, then there exists one third stage equilibrium in which R_A is the only one who buys, and one in which R_B is the only one who buys, but none in which both buy. If one downstream firm has a contract with an exclusivity clause, this downstream firm will buy from M if and only

[8] That $\pi_i(w_i, w_j) - \pi_i(\infty, w_j) < \pi_i(w_i, \infty) - \pi_i(\infty, \infty)$ follows from Assumption 1.

if $\pi_i(w_i, \infty) - \pi_i(\infty, \infty) \geq F_i$.[9] The rival will then, given that it is not protected by an exclusivity clause, buy if and only if $\pi_j(w_j, w_i) - \pi_j(\infty, w_i) \geq F_j$.

When contracts can be made contingent on exclusivity, it is straightforward to show that the joint profit of M and R_i must be at least Π_i^m in any equilibrium. If not, R_i could profitably deviate by offering an an exclusivity-contingent contract in stage (i) with $w_i = w_i^m$.

Lemma 2. When contracts can be made contingent on exclusivity, the joint profit of M and R_i must be at least Π_i^m in any equilibria.

Common agency equilibria Suppose there exists a common agent equilibrium, i.e., an equilibrium in which both downstream firms buy strictly positive quantities from M in stage (iii), in which M has accepted the contracts $\tilde{T}_A = (\tilde{S}_A, \tilde{w}_A, \tilde{F}_A)$ and $\tilde{T}_B = (\tilde{S}_B, \tilde{w}_B, \tilde{F}_B)$. M may have been offered a menu of tariffs, some of which may have been contingent on exclusivity, but $\tilde{T}_A$ and $\tilde{T}_B$ are the tariffs accepted. Since R_i has chosen to buy from M in stage (iii) rather than only using the other available inputs, it must be the case that the following holds.

$$\tilde{F}_i \leq \pi_i(\tilde{w}_i, \tilde{w}_j) - \pi_i(\infty, \tilde{w}_j). \tag{1}$$

If the inequality in (1) does not bind, $\tilde{w}_j$ has to maximize the joint profit of M and R_j given $\tilde{w}_i$. Otherwise, R_j could profitably deviate by changing w_j in a way that increased the joint flow profit of M and R_j, without affecting the fixed fees paid by R_i to M. That is, if the inequality does not hold with equality, we need that $\tilde{w}_j = W_j(\tilde{w}_i)$.

If (1) holds with equality, F_i is equal to the gains R_i has from buying from M in stage (iii), and since F_i is conditional on trade, R_i is protected from opportunistic behavior from the rival. If the rival tries to free-ride on the downstream margin of R_i by slightly reducing w_j in stage (i), R_i will choose not to buy in stage (iii) and avoid paying the fixed fee. A common agency equilibrium must however also be robust to possible deviations to exclusivity, in which one of the downstream firms offers a contract that induces a third stage equilibrium where only this downstream firm buys from M. The following lemma will prove useful.

Lemma 3. In any equilibrium in which the downstream firms offer $(\tilde{S}_A, \tilde{w}_A, \tilde{F}_A)$ and $(\tilde{S}_B, \tilde{w}_B, \tilde{F}_B)$, M accepts these tariffs, both downstream firms buy from M in stage (iii),

[9] Remember that if M also sells to the rival in stage (iii), a firm with an exclusivity clause will be exempt from paying M.

and where $\tilde{w}_j > W_j(\tilde{w}_i)$, the joint profit of R_j and M must be at least $\Pi_j^m + \max\{0, \tilde{S}_i\}$.

Proof. See Appendix.

Since the inequality in (1) must bind if $\tilde{w}_j > W_j(\tilde{w}_i)$, R_i's profit in a common agency equilibrium with $(\tilde{T}_A, \tilde{T}_B)$ such that $\tilde{w}_j > W_j(\tilde{w}_i)$ is $\pi_i(\infty, \tilde{w}_j) - \tilde{S}_j$. Since the joint profit of R_j and M in any such equilibrium is at least $\Pi_j^m + \max\{0, \tilde{S}_i\}$, and since the industry profit is $\Pi(\tilde{w}_A, \tilde{w}_B)$, we must have that $\Pi(\tilde{w}_A, \tilde{w}_B) - (\pi_i(\infty, \tilde{w}_j) - \tilde{S}_i) \geq \Pi_j^m + \max\{0, \tilde{S}_i\}$. From this condition we get the following proposition.

Proposition 1. If $w_j > W_j(w_i)$ for a $j \in \{A, B\}$ and $\Pi_j^m + \pi_i(\infty, w_j) > \Pi(w_A, w_B)$, there can be no common agency equilibria with three-part tariffs and wholesale prices (w_A, w_B).

The condition under which no common agency equilibria exists may be satisfied for every (w_A, w_B) such that $\Pi(w_A, w_B) > \Pi(W_A(w_B), W_B(w_A))$. Even though $\Pi_j^m + \pi_i(\infty, w_j^m) < \Pi(w_A, w_B)$ holds for $(w_A, w_B) = (w_A^*, w_B^*)$ by Assumption 2, we also have that $\pi_i(\infty, w_j^*) > \pi_i(\infty, w_j^m)$ whenever $w_j^* > w_j^m$.

What is the intuition for Proposition 1? Since the downstream firms have the opportunity to use several inputs, they will in stage (iii) typically have an alternative to buying a positive quantity from M that gives them a positive flow profit. Importantly, the value of this outside option depends on the contract between the rival downstream firm and M. The value if the rival has a contract intended for common agency, where the wholesale prices typically is above marginal cost in order to dampen downstream competition, is generally greater than the value when the rival has a contract intended for an exclusive relationship with M, in which case wholesale price is typically equal to marginal cost.[10] The flow profit of R_i in any equilibrium is bounded below by the value of its alternative in stage (iii). This flow profit might however be greater than R_i's "contribution" to the industry profit, in which case it will be profitable for R_j to induce an exclusive relationship with M.

When is $\Pi_j^m + \pi_i(\infty, w^*) > \Pi^*$ most likely to hold, making profit-maximizing common agency equilibria infeasible? Obviously, if the downstream firms are sufficiently differentiated, Π^* will be substantially greater than Π_j^m for $j = A, B$, and $\pi_i(\infty, w_j^m)$

[10]This would not be the case with single-input downstream firms, because then the flow profit a given downstream firms can achieve by not buying from M is zero, regardless of whether the rival buys or not.

close to $\pi_i(\infty, w_j^*)$, and the condition will not hold, so a reasonable amount of downstream substitutability is needed. For a given level of differentiation, weaker downstream competition would reduce the scope for common agent equilibria. For example, the condition is more likely to hold, and thereby rule out profit maximizing common agency equilibria, when downstream firms compete in quantities than when they compete in prices. If the downstream firms are close to perfect substitutes, $\Pi_j^m + \pi_i(\infty, w_j^m)$ approaches Π^*, and if they compete in quantities, $\pi_i(\infty, w_j^*)$ will generally be substantially greater than $\pi_i(\infty, w_j^m)$, which would tend to make $\Pi_j^m + \pi_i(\infty, w^*) > \Pi^*$ hold.

Proposition 1 may rule out the existence of common agency equilibria with $(w_A, w_B) \neq (W_A(w_B), W_B(w_A))$. Since the joint profit of R_i and M is bounded below by Π_i^m by Lemma 2, it follows that there might not exist any common agency equilibria if $\Pi(W_A(w_B), W_B(w_A)) < \max\{\Pi_A^m, \Pi_B^m\}$. The latter condition will be satisfied when the downstream firms are not too differentiated.

Exclusionary equilibria If the contracts can include exclusivity clauses, there always exists exclusionary equilibria. To see this, note that when M accepts a exclusivity-contingent tariff from a downstream firm, this firm need not worry about the rival free-riding on its margin. This enables a given downstream firms to offer M its entire gain from being the exclusive downstream firm. This guarantees the existence of (exclusionary) equilibria in the game. The following proposition specifies an exclusionary equilibrium with exclusivity-contingent two-part tariffs.

Proposition 2. Let $\Pi_i^m - \pi_i(\infty, w_j^m) \geq \Pi_j^m - \pi_j(\infty, w_i^m)$, and consider exclusivity-contingent contracts, where

- $(S_i, w_i, F_i) = (\pi_i(w_i^m, \infty) - \pi_j(\infty, w_i^m) - (\Pi_i^m - \Pi_j^m), w_i^m, 0)$
- $(S_j, w_j, F_j) = (\pi_j(w_j^m, \infty) - \pi_j(\infty, w_i^m), w_j^m, 0)$

There exists an equilibrium in which the downstream firms offer these contracts, M accepts only the contract from R_i and R_i buys from M in stage (iii). The profit of M is $\Pi_j^m - \pi_j(\infty, w_i^m)$, the profit of R_i is $\pi_j(\infty, w_i^m) + (\Pi_i^m - \Pi_j^m)$, the profit of R_j is $\pi_j(\infty, w_i^m)$. No other exclusionary equilibrium leaves the downstream firms with higher profits. If $\Pi_i^m - \pi_i(\infty, w_j^m) > \Pi_j^m - \pi_j(\infty, w_i^m)$, R_i buys from M in all exclusionary equilibria. If $\Pi_i^m - \pi_i(\infty, w_j^m) = \Pi_j^m - \pi_j(\infty, w_i^m)$, there exists equilibria in which only R_i buys and equilibria in which only R_j buys.

Proof. See Appendix.

If M were not in the industry, R_j's profit would be $\pi_j(\infty, \infty)$. It follows from Lemma 1 that this firm's profit is lower in any exclusionary equilibrium than it would have been if M was not present. R_i may also lose from the interaction with M. If $\pi_j(\infty, w_i^m) + (\Pi_i^m - \Pi_j^m) < \pi_i(\infty, \infty)$, R_i's profit in any exclusionary equilibrium is less than what it would make if M was not present in the industry. This last condition will hold if the downstream firms are sufficiently symmetric.

What is the intuition behind these results? In an equilibrium where only one downstream firm purchases from M, the excluded downstream firm can do no better than to offer M its entire gain from not being excluded. This gain is the difference between its flow profit when being the exclusive downstream firm and its profit when the rival is the exclusive downstream firm. Letting the rival end up as exclusive dealer not only makes a downstream firm miss out on increased revenue made possible by the use of Product 1, it may also reduce the profitability of using the other available inputs, compared to a situation in which neither downstream firm trades with M. This second effect, which obviously is not present with single-input downstream firms, might intensify the bidding for exclusivity to such a degree that both downstream firms end up with negative payoff from the interaction with the manufacturer; M's power to decide which firm is the exclusive downstream firm may enable her to extract more of the industry profit than her contribution to it. Both downstream firms would then have preferred that M stayed out of the market, even though the manufacturer offers a profitable product and the downstream firms have all the bargaining power.

Secret contracts We have seen that that there might not exist common agent equilibria when public contract offers are allowed to be contingent on exclusivity. It should be noted that the probability of there existing common agent equilibria would be further reduced if the downstream firms could not observe the details of the contract between M and the rival before stage (iii). This would have invited opportunistic behavior: A downstream firm and the manufacturer would have had an incentive to secretly negotiate a reduction in the input price, and free ride on any downstream margin of the other downstream firm, thereby increasing their joint profit. This would limit the profit attainable through common agency, and would increase the probability of profitable deviations to exclusive relationships. The exclusionary equilibria supported by exclusivity clauses would still exist, as long as the downstream firms could observe whether M actually traded with the rival or not.

4 The manufacturer offer contracts

When a monopolist manufacturer offers contracts to single-input downstream firms, simple two-part tariffs lets the manufacturer induce an equilibrium where industry profit is maximized, and at the same time capture the entire profit through the fixed fees. The equilibrium outcome is not affected by allowing the manufacturer to offer more complex contracts. In our setting with multi-input downstream firms, however, the complexity of the contracts available to M may affect not only the profits of the firms, but also whether both downstream firms trade with M in equilibrium and the equilibrium input prices.

Contracts with single two-part tariffs Consider the game specified in Section 3, but assume that M is the one offering contracts in stage (i), and that R_A and R_B accepts or rejects these contracts in stage (ii). Unless noted, it will be assumed that M is not able to make the contract offers explicitly contingent on exclusivity.

Suppose first that M is restricted to offering the downstream firms contracts with single two-part tariff with non-contingent fixed fees, i.e., contracts on the form $T_i = (S_i, w_i)$.[11] Let us consider the properties of any non-exclusionary equilibrium with wholesale prices (w_A, w_B) in this setting. If R_i is to accept a tariff in stage (ii), S_i must be such that R_i is left with at least $\pi_i(\infty, w_j)$. The equilibrium profit of M in any non-exclusionary equilibrium with wholesale prices (w_A, w_B) is therefore given by $\Pi(w_A, w_B) - \pi_A(\infty, w_B) - \pi_B(\infty, w_A)$. The profit of M in a non-exclusionary equilibrium is thus bounded above by $\Pi(\hat{w}_A, \hat{w}_B) - \pi_A(\infty, \hat{w}_B) - \pi_B(\infty, \hat{w}_A)$, where $\hat{w}_i \equiv \arg\max_{w_i} \Pi(w_i, w_j) - \pi_i(\infty, w_j) - \pi_j(\infty, w_i)$. Note that when $(w_A, w_B) = (w_A^*, w_B^*)$, a slight reduction in, say w_A, gives only a second order reduction in $\Pi(w_A, w_B)$, but a first order reduction in $\pi_B(\infty, w_A)$. This means that $(\hat{w}_A, \hat{w}_B) \neq (w_A^*, w_B^*)$.[12]

If M sells only to one downstream firm, say R_i, this firm must be left with a profit of at least $\pi_i(\infty, \infty)$ in any equilibrium. If not, it could profitably deviate by not

[11] The results of this subsection holds whether or not we allow M to offer menus of contracts or not, as long as the downstream firms chooses between these contracts in stage (ii) and each contract consists of a single two-part tariff.

[12] In this framework, the contract between R_i and M affects the profit of R_j even when R_j does not trade with M. The result that contract externalities affecting non-traders can result in distortions is established in Segal (1999).

accepting the contract in stage (ii). The best M can achieve through selling only to R_i is then $\Pi_i^m - \pi_i(\infty,\infty)$. Note that this amount is bounded above by $\Pi(\hat{w}_A,\hat{w}_B) - \pi_A(\infty,\hat{w}_B) - \pi_B(\infty,\hat{w}_A)$. We get the following proposition

Proposition 3. When restricted to offering two-part tariffs, equilibrium wholesale prices will be equal to $(\hat{w}_A,\hat{w}_B) \neq (w_A^*, w_B^*)$, where $\hat{w}_i \equiv \arg\max_{w_i} \Pi(w_i,w_j) - \pi_i(\infty,w_j) - \pi_j(\infty,w_i)$. The equilibrium profit of M is $\Pi(\hat{w}_A,\hat{w}_B) - \pi_A(\infty,\hat{w}_B) - \pi_B(\infty,\hat{w}_A)$. The equilibrium profit of R_i is $\pi_i(\infty,\hat{w}_j)$.

Contracts with single three-part tariffs Assume now that M can include contingent fixed fees in the contract.[13] This gives us three-part tariffs of the following form: $T_i = (S_i, w_i, F_i)$. Note first that, as was the case with two-part tariffs, in any non-exclusionary equilibrium with wholesale prices (w_A, w_B), R_i must be left with at least $\pi_i(\infty, w_j)$, since it otherwise could profitably deviate by not accepting the offered contract in stage (ii). This means that the profit of M in any non-exclusionary equilibrium is bounded above by $\Pi(\hat{w}_i,\hat{w}_j) - \pi_i(\infty,\hat{w}_j) - \pi_j(\infty,\hat{w}_i)$, where again $\hat{w}_i \equiv \arg\max_{w_i} \Pi(w_i,w_j) - \pi_i(\infty,w_j) - \pi_j(\infty,w_i)$.

Let us now consider possible equilibria of this game. First, let $\underline{w}_j$ be the wholesale price for R_j that minimizes the flow profit of R_i, when only R_j is buying from M. That is let, $\underline{w}_j \equiv \arg\min_{w_j \geq 0} \pi_i(\infty, w_j)$. The assumptions made in Section 2 imply that $\underline{w}_j = 0$. The possibility of making fixed fees contingent on trade makes non-exclusionary candidate equilibria more susceptible to deviations to exclusivity. By offering the one downstream firm, say R_j, a contract that is such that it will choose to buy if and only if the rival rejects its contract, M is able to reduce the outside option of the other downstream firm, R_i, and therefore extract a bigger share of their joint profit, Π_i^m. We get the following proposition.

Proposition 4. If $\Pi(\hat{w}_A,\hat{w}_B) - \pi_A(\infty,\hat{w}_B) - \pi_B(\infty,\hat{w}_A) < \max\{\Pi_A^m - \pi_A(\infty,\underline{w}_B), \Pi_B^m - \pi_B(\infty,\underline{w}_A)\}$, there exists no common agent equilibria if M is restricted to offer the downstream firms three-part tariffs.

Proof. See appendix.

If the condition in Proposition 4 is satisfied, there will be exclusion in all equilibria. We get the following proposition.

[13] As was the case with two-part tariffs, the results in this subsection holds whether or not M can offer menus of contracts.

Proposition 5. Suppose $\Pi_i^m - \pi_i(\infty, \underline{w}_j) \geq \Pi_j^m - \pi_j(\infty, \underline{w}_i)$. Then if $\Pi(\hat{w}_A, \hat{w}_B) - \pi_A(\infty, \hat{w}_B) - \pi_B(\infty, \hat{w}_A) \leq \Pi_i^m - \pi_i(\infty, \underline{w}_j)$, there will be an exclusionary equilibrium in which M offers R_j the contract $(S_j, w_j, F_j) = (0, \underline{w}_j, \pi_j(\underline{w}_j, \infty) - \pi_j(\infty, \infty))$ and R_i the contract $(S_i, w_i, F_i) = (\pi_i(w_i^m, \infty) - \pi_i(\infty, \underline{w}_j), w_i^m, 0)$, both downstream firms accept their contracts, and only R_i buys from M in stage (iii). In this equilibrium R_j gets $\pi_j(\infty, w_i^m)$, R_i gets $\pi_i(\infty, \underline{w}_j)$ and M gets $\Pi_i^m - \pi_i(\infty, \underline{w}_j)$.

Proof. See appendix.

By being able to include contingent fixed fees in the contracts offered the downstream firms, M is able to make the acceptance decision of one downstream firm decide who ends up as the exclusive downstream firm. In the equilibrium specified in Proposition 5, M offers the excluded downstream firm R_j a contract that is such that it will buy from M, and flood the market, if and only if the rival rejects its contract offer in stage (ii). This makes it costly for the R_i to reject the contract offered to it by M, since this would make the rival a *de facto* exclusive downstream firm. As a consequence, M can extract more of Π_i^m than it was able to when only being able to offer two-part tariffs, and an exclusion is more likely to be the equilibrium outcome of the game.

Contracts with menus of two-part tariffs As we have seen, the scope for non-exclusionary equilibria can be limited when we restrict M to offer contracts with single tariffs. Assume now that M can offer the downstream firms contracts that include a menu of two-part tariffs to choose from in stage (iii). Then, there always exists a non-exclusionary equilibrium that gives M a higher profit than what she is able to achieve in any exclusionary equilibrium.[14] This non-exclusionary equilibrium can be supported by a strategy that is similar to the one used to induce exclusion in Proposition 5, in the sense that both strategies use the contract offered one downstream firm to reduce the outside option of the rival downstream firm.

Suppose that M can offer R_i a contract that gives it the choice between two two-part tariffs in stage (iii), that is a contract of the form $(S_i, w_i^E, w_i^C, F_i^E, F_i^C)$, where in stage (iii), R_i can choose between buying at w_i^E and paying a fixed fee of F_i^E, and buying at w_i^C and paying a fixed fee of F_i^C.[15] This enables M to induce the common agent equilibrium specified in the following proposition.

[14] Alternatively, one could let M offer the downstream firms a menu of quantity-forcing tariffs to choose from in stage (iii).

[15] The firms can no longer avoid paying a fixed fee by choosing not to buy in stage (iii).

Proposition 6. There exists a non-exclusionary equilibrium in which M offer R_i ($i = A, B$) the following contract.

- $S_i = -\pi_i(\infty, \underline{w}_j)$
- $F_i^E = \pi_i(\underline{w}_i, w_j^*)$, and $F_i^C = \pi_i(w_i^*, w_j^*)$
- $w_i^C = w_i^*$ and $w_i^E = \underline{w}_i$

In the equilibrium both downstream firms accept their contract offers, and choose to buy at w_i^C (and pay F_i^C) in stage (iii). The profit of M is $\Pi^* - \pi_A(\infty, \underline{w}_B) - \pi_B(\infty, \underline{w}_A)$, the profit of R_A is $\pi_A(\infty, \underline{w}_B)$, and the profit of R_B is $\pi_B(\infty, \underline{w}_A)$.

Proof. See Appendix.

The contracts offered in this equilibrium is such that in third-stage game equilibrium, both downstream firms will choose the tariff designed for common equilibrium if both accept their contract offers in stage (iii), whereas they will choose the tariffs designed for exclusivity if the rival rejects its contract offer in stage (ii). The fact that the contracts are "incentive compatible" in this way makes it so costly for the downstream firms to reject the contract offers from M in stage (ii) that they both are willing to accept a contract – and induce a common agent equilibrium in stage (iii) – even if this only leaves them with the smallest conceivable profit (in the absence of restrictions on resale prices or quantities), $\pi_i(\infty, \underline{w}_j)$ for $i = A, B$.

The equilibrium specified in Proposition 6 might however not be a perfectly coalition-proof Nash equilibrium (PCPNE), in the sense of Bernheim et al. (1987). Clearly, both downstream firms would be better off if they both rejected the offer from M in stage (ii), which would leave R_i with $\pi_i(\infty, \infty)$. Such a joint deviation would be self-enforcing if and only if $\pi_i(\underline{w}_i, \infty) - S_i - F_i^E \leq \pi_i(\infty, \infty)$ for $i = A, B$. From this, we get the following proposition.

Proposition 7. The equilibrium specified in Proposition 6 is a PCPNE if and only if $\pi_i(\underline{w}_i, \infty) - \pi_i(\underline{w}_i, w_j^*) > \pi_i(\infty, \infty) - \pi_i(\infty, \underline{w}_j)$ for a $i \in \{A, B\}$.

If, however, M can make contracts contingent on exclusivity, she need not worry about joint deviations from the retailers. The following proposition specifies an PCPNE using contracts that are contingent on exclusivity that guarantees M a profit equal to the one specified in Proposition 6.

Proposition 8. Let M offer R_i a contract $(S_i, w_i^E, w_i^C, F_i^E, F_i^C)$, where (w_i^E, F_i^E) is a two-part tariff that is valid if and only of R_j rejects an offer from M, and (w_i^C, F_i^C) a tariff that is valid if and only if R_j accepts an offer from M. There then exists a non-exclusionary PCPNE in which M offer R_i $(i = A, B)$ a contract such that:

- $S_i = -\pi_i(\infty, \underline{w}_j)$
- $F_i^E = 0$, and $F_i^C = \pi_i(w_i^*, w_j^*)$
- $w_i^C = w_i^*$ and $w_i^E = \underline{w}_i$

In the equilibrium both downstream firms accept their contract offers, and therefore buy at w_i^C (and pay F_i^C) in stage (iii). The profit of M is $\Pi^* - \pi_A(\infty, \underline{w}_B) - \pi_B(\infty, \underline{w}_A)$, the profit of R_A is $\pi_A(\infty, \underline{w}_B)$, and the profit of R_B is $\pi_B(\infty, \underline{w}_A)$.

Proof. See appendix.

From a consumer welfare perspective, the effect of allowing M to offer more complex contracts is ambiguous. When restricted to offering simple two part tariffs, one expects both downstream firms to buy from M at wholesale prices below the profit maximizing level. Allowing three-part tariffs increases the probability of there being exclusion in equilibrium, with the effect on prices being ambiguous. Allowing M to offer contracts with menus of tariffs secures the existence of a non-exclusionary equilibrium in which the downstream firms' rents are minimized, and the wholesale prices are at the profit maximizing level. This equilibrium might however be susceptible to joint deviations from the downstream firms. Allowing contracts that are contingent on exclusivity guarantees the presence of M's product at both downstream firms (even when allowing for joint deviations), but also that wholesale prices are set to fully dampen downstream competition.

5 Buyer cooperation

As we have seen in the last two sections, the downstream firms are often not able to gain from trading with M; Their profit is often reduced by M's presence in the market. The precarious position of the downstream firms begs the following question: What would the downstream firms gain from cooperating when interacting with the manufacturer?

The formation of buyer groups, understood broadly as a group of separate firms that cooperate as buyers in the input market but compete as sellers in the output market, can make the buyer concentration in the upstream market greater than the seller concentration in the downstream market. Forms of buyer cooperation are commonplace among groceries and pharmacists, but exists also among TV-distributors and even airlines (Dobson and Waterson 1999; Dobson 2003; Dana Jr 2012). The competitive effects of downstream firms cooperating in input markets are subject to significant recent debate. Increased buyer power might be beneficial for end-consumers if it is used to obtain lower input prices, which again tends to reduce prices in the downstream market (Dana Jr 2012; Marvel and Yang 2008). That buyer groups can be used to dampen competition in the downstream market, is however also well known (Foros and Kind 2008).

In the framework of this article, the presence of the common manufacturer often hurt the downstream firms. This implies that they may gain from cooperating when interacting with the upstream firm, even when this cooperation neither lets them extract a larger part of the profit this firm brings to the industry, nor dampen competition in the downstream market, but simply exclude the upstream firm from the market altogether.

One form of buyer cooperation is the ability to make joint listing decisions, that is, when a group of buyers can commit to a decision of which suppliers they are allowed to trade with. In the framework of this article it is clear that the downstream firms could have an incentive to agree to delist M. In many of the equilibria discussed in this article, both downstream firms are made worse off by the presence of M. In these equilibria the downstream firms would be better off if they both refrained from trading with M, but it is a dominant strategy for both to trade with M. Committing not to trade with M then clearly makes both downstream firms better off.[16]

From a consumer welfare perspective, allowing the downstream firms the opportunity to jointly delist M seems unnecessary at best. Buyer cooperation do however often entail more than making joint listing decisions. Several authors have examined the effect of letting downstream firm interact with their suppliers through a common

[16] Caprice and Rey (2012) model the joint listing decision in a slightly different way. In their model competing downstream firms simultaneously get contract offers from a common supplier. If one downstream firm rejects its offer, the other downstream firms are prohibited from trading with the supplier. It is easy to show that if such a (public) veto-rule is implemented in the framework of this article, the profit of R_i in any equilibrium in which M offer contracts would be bounded below by $\pi_i(\infty, \infty)$.

agent (Foros and Kind 2008; Dana Jr 2012; Marvel and Yang 2008). If one allowed the downstream firms to interact with M through such a common agent and assumed that this common agent seeked to maximize the profit of the two downstream firms, one would expect both downstream firms to buy from M in any equilibrium. The effect on consumer welfare of allowing such cooperation is ambiguous: It could increase the downstream availability of M's product, while the effect on downstream prices is uncertain.

6 Conclusion

The interplay between the division of bargaining power and the complexity of supply contracts has been central to several recent articles in the vertical contracting literature. The present article contributes to this literature by studying a framework with two competing downstream firms and a common supplier, and assuming that the downstream firms have several sources of input.

When competing single-input downstream firms offer contracts to a common supplier, the complexity of the contracts affects whether there is exclusion in equilibrium. When restricted to offering single-three part tariffs, the strongest downstream firm will be able to exclude its rival in all equilibria (Marx and Shaffer 2007). When the downstream firms can offer menus of contracts, non-exclusionary equilibria in which total industry profit is maximized are sustainable (Miklós-Thal et al. 2011; Rey and Whinston 2013). The present article shows that with multi-input downstream firms, the profit maximizing outcome is not necessarily sustainable, even when the downstream firms can offer menus of contracts, and even when these contracts can be made contingent on exclusivity. In exclusionary equilibria – which sometimes are the only equilibria of the game – both downstream firms are often made worse off by the presence of the common manufacturer.

Complex supply contracts are often associated with buyer power. When a common upstream manufacturer offers public contracts to two competing single-input downstream firms, simple two-part tariffs are sufficient to maximize the industry profit and extract all rent from the downstream firms. This article shows that when the downstream firms have multiple sources of inputs, this is no longer the case. The common manufacturer will generally find it profitable to offer more complex contracts than simple two-part tariffs (sometimes even contracts involving exclusive territory clauses), and the range of contracts available to the manufacturer affects the equilibrium outcome in several ways: How profits are divided, whether both downstream firm trades with the common manufacturer, and at which wholesale prices trade occurs.

The article also illustrates that allowing the rival to be alone in trading with the common manufacturer is often very costly for the downstream firms. This often makes them offer, or accept, contracts that makes them worse off than they would have been if the common manufacturer was not in the market. The downstream firms would then profit if they were able to jointly commit to not trade with the common manufacturer.

Appendix

Proof of Lemma 3. Assume that both downstream firms buy from M in stage (iii), at wholesale prices $(\tilde{w}_A, \tilde{w}_B)$, where $\tilde{w}_j > W_j(\tilde{w}_i)$. Let Π_j and Π_M be the equilibrium profits of R_j and M, and suppose that $\Pi_j + \Pi_M < \Pi_j^m + \max\{0, \tilde{S}_i\}$.

Note that M cannot earn more than Π_M by only accepting the offer from R_i or rejecting both contract offers, since if it did it would have a profitable deviation. Now, suppose R_j deviates by offering a exclusivity-contingent contract $(S_j, w_j, F_j) = (X, w_j^m, 0)$, where $X \in (\Pi_M - \pi_M(w_j^m, \infty) - \max\{0, \tilde{S}_i\}, \pi_j(w_j^m, \infty) - \Pi_j)$.

Since F_j is zero, R_j will always buy from M in stage (iii) if this contract offer is accepted. Since $\tilde{w}_j > W_i(\tilde{w}_i)$, we must have that $\tilde{F}_i = \pi_i(\tilde{w}_i, \tilde{w}_j) - \pi_i(\infty, \tilde{w}_j)$. This means that if M accepts R_j's deviation offer *and* accepts the equilibrium tariff $\tilde{T}_i$ from R_i, R_i will not buy in stage (iii) if $\tilde{F}_i = \pi_i(\tilde{w}_i, \tilde{w}_j) - \pi_i(\infty, \tilde{w}_j) > \pi_i(\tilde{w}_i, w_j^m) - \pi_i(\infty, w_j^m)$, or equivalently, if

$$
\begin{aligned}
(\pi_i(\infty, \tilde{w}_j) - \pi_i(\infty, w_j^m)) - ((\pi_i(\tilde{w}_i, \tilde{w}_j) - \pi_i(\tilde{w}_i, w_j^m)) &\equiv \int_{w_j^m}^{\tilde{w}_j} \int_{\tilde{w}_i}^{\infty} \frac{\partial^2 \pi_i(w_i, w_j)}{\partial w_j \partial w_i} \, \mathrm{d}w_i \mathrm{d}w_j \\
&< 0,
\end{aligned}
$$

which holds by Assumption 1, since $w_j^m \leq W_j(\tilde{w}_i) < \tilde{w}_j$.

By accepting the deviation offer from R_j and the equilibrium offer from R_i, M gets a profit of $\pi_M(w_j^m, \infty) + X + \tilde{S}_i > \Pi_M + \tilde{S}_i - \max\{0, \tilde{S}_i\}$. By accepting only deviation offer from R_i, M gets $\pi_M(w_j^m, \infty) + X > \Pi_M - \max\{0, \tilde{S}_i\}$. Note that when $\tilde{S}_i$ is positive, M always gets a higher profit from accepting R_j's deviation offer and R_i's equilibrium offer than accepting no contracts or only a contract (either the equilibrium contract or another contract) from R_i. When $\tilde{S}_i$ is negative, M always gets a higher profit from accepting only R_j's deviation offer than accepting no contract or only a contract from R_i. If M accepts only the deviation offer or the deviation offer and the equilibrium offer from R_i, R_j is secured $\pi_j(w_j^m, \infty) - X > \Pi_j$.

If M accepts the deviation offer from R_j and a contract from R_j, say $\hat{T}_i$, that is not the equilibrium contract, there are two cases to consider. If R_i buys in the third stage, then M will not receive a payment from R_j in stage (iii). The profit of M is then bounded above by what she would get by only accepting $\hat{T}_i$, which again is bounded above by Π_M, so this can not be a continuation equilibrium. If R_i does not buy in stage (iii), R_j gets $\pi_j(w_j^m, \infty) - X > \Pi_j$. This establishes that R_j's profit in all continuation equilibria is above Π_j, and that the deviation is profitable.

Q.E.D.

Proof of Proposition 2. Consider first M's acceptance decision. If M deviates and rejects both offers she gets a profit of zero, if she deviates and accepts only the offer from R_j she gets $\Pi_j^m - \pi_j(\infty, w_i^m)$, which is exactly what she gets in equilibrium. If she deviates and accepts both offers, the downstream firms will not pay anything to M in stage (iii), and M's profit is weakly negative. Hence, M has no profitable deviation.

R_i has no profitable deviation. M can get $\Pi_j^m - \pi_j(\infty, w_i^m)$ by accepting R_j's contract offer. If R_i deviates and offers a different contract in stage (i), M must therefore be left with at least $\Pi_j^m - \pi_j(\infty, w_i^m)$ if she is to accept this offer. Since the joint profit of M and R_i when R_i buys from M is bounded above by Π_i^m, the profit of R_i following any deviation in stage (i) is bounded above by $(\Pi_i^m - \Pi_j^m) + \pi_j(\infty, w_i^m)$, which is exactly what it gets in equilibrium.

Finally, R_j has no profitable deviation. M gets $\Pi_j^m - \pi_j(\infty, w_i^m)$ by accepting R_i's offer. Suppose R_j deviates and offers a different contract in stage (i). If M is to accept this contract, she must be left with at least $\Pi_j^m - \pi_j(\infty, w_i^m)$. The joint profit of M and R_j if M accepts the deviation offer from R_j is bounded above by Π_j^m. This bounds the profit of R_j following any deviation from above at $\pi_j(\infty, w_i^m)$, which is exactly what it gets in equilibrium.

The rest of the proof establishes that no other exclusionary equilibrium can leave either downstream firm with higher profits than the equilibrium specified in Proposition 2. Suppose that the downstream firms can offer M exclusivity-contingent three-part tariffs, and that we are in an exclusionary equilibrium in which R_i buys from M in stage (iii) at wholesale price $\tilde{w}_i$. The profit of the excluded R_j must in any such equilibrium be $\pi_j(\infty, \tilde{w}_i)$. If it was more than this, M could profitably deviate by not accepting R_j's contract offer in stage (ii), it it was less, R_j could profitably deviate by not offering a contract in stage (i). This implies that $\tilde{w}_i = w_i^m$. If this was not the case, R_i could profitably deviate by offering an exclusivity-contingent contract with $w_i = w_i^m$, since this would increase the joint profit of R_i and M.

Suppose now that we are in an exclusionary equilibrium in which R_j earns more than $\pi_j(\infty, w_i^m)$. Since $\Pi_i^m - \pi_i(\infty, w_j^m) \geq \Pi_j^m - \pi_j(\infty, w_i^m)$, the industry profit in any exclusionary equilibrium is bounded above by $\Pi_i^m + \pi_j(\infty, w_i^m)$. This means that the joint profit of R_i and M is strictly less than Π_i^m. Denote the equilibrium profits of M and R_i by Π_i and Π_M and suppose that R_i deviates by offering an exclusive-contingent contract in stage (i) with $(S_i, w_i, F_i) = (X, w_i^m, 0)$, where $X \in (\Pi_M - \pi_M(w_i^m, \infty), \pi_i(w_i^m, \infty) - \Pi_i)$. By only accepting this deviation offer, M earns $\pi_M(w_i^m, \infty) + X > \Pi_M$. If M accepts the deviation offer and an offer, say $\tilde{T}_j$ from R_j, there are two cases to consider. If R_j does not buy in stage (iii), then the profit of R_i is $\pi_i(w_i^m, \infty) - X > \Pi_i$. If R_j does buy in stage (iii), the profit of M is bounded above by what she could get by only accepting $\tilde{T}_j$, which is bounded above by Π_M, so this cannot be a continuation equilibrium. The profit of R_i is then $\pi_i(w_i^m, \infty) - X > \Pi_i$ in any continuation equilibrium, so the deviation is profitable.

Suppose now that we are in an exclusionary equilibrium in which the profit of R_i is greater than $(\Pi_i^m - \Pi_j^m) + \pi_j(\infty, w_i^m)$. Since the industry profit is bounded above by $\Pi_i^m + \pi_j(\infty, w_i^m)$, the joint profit of R_j and M must be strictly less than Π_j^m. By similar reasoning as above, R_j would then have a profitable deviation in which it offers an exclusivity-contingent contract with $(S_j, w_j, F_j) = (X, w_j^m, 0)$, where $X \in (\Pi_M - \pi_M(w_j^m, \infty), \pi_j(w_j^m, \infty) - \Pi_j)$.

Q.E.D.

Proof of Proposition 4. Suppose we are in a common agent equilibrium with wholesale prices $(\tilde{w}_A, \tilde{w}_B)$ and that $\Pi(\hat{w}_A, \hat{w}_B) - \pi_A(\infty, \hat{w}_B) - \pi_B(\infty, \hat{w}_A) < \max\{\Pi_A^m - \pi_A(\infty, \underline{w}_B), \Pi_B^m - \pi_B(\infty, \underline{w}_A)\}$. The profit of R_i in this equilibrium must be at least $\pi_i(\infty, \tilde{w}_j)$, if not it could profitably deviate by rejecting the contract offer in stage (ii). It follows that the equilibrium profit of M is bounded above by $\Pi(\tilde{w}_A, \tilde{w}_B) - \pi_A(\infty, \tilde{w}_B) - \pi_B(\infty, \tilde{w}_A) \leq \Pi(\hat{w}_A, \hat{w}_B) - \pi_A(\infty, \hat{w}_B) - \pi_B(\infty, \hat{w}_A) < \max\{\Pi_A^m -$

$\pi_A(\infty, \underline{w}_B), \Pi_B^m - \pi_B(\infty, \underline{w}_A)\}$.

Let $\Pi_A^m - \pi_A(\infty, \underline{w}_B) \geq \Pi_B^m - \pi_B(\infty, \underline{w}_A)$, and suppose M deviates by offering the contracts $(S_A, w_A, F_A) = (\pi_A(w_A^m, \infty) - \pi_A(\infty, \underline{w}_B) - \epsilon, w_A^m, 0)$ and $(S_B, w_B, F_B) = (-\epsilon, \underline{w}_B, \pi_B(\underline{w}_B, \infty) - \pi_B(\infty, \infty))$, where $\epsilon \in (0, (\Pi_A^m - \pi_A(\infty, \underline{w}_B) - \Pi(\tilde{w}_A, \tilde{w}_B) + \pi_A(\infty, \tilde{w}_B) + \pi_B(\infty, \tilde{w}_A))/2)$.

If R_A accepts its contract in stage (ii), then it will buy from M in stage (iii) in any continuation equilibrium. Since S_B is negative, R_B will accept the contract offer from M in stage (ii) any continuation equilibrium following the deviation by M. R_B will buy in stage (iii) if and only if R_A has not accepted its contract in stage (ii). This means that R_A gets $\pi_A(\infty, \underline{w}_B) + \epsilon$ by accepting its contract in stage (ii), and $\pi_A(\infty, \underline{w}_B)$ by rejecting it. This means that both downstream firms accept their offers following the deviation, and only R_A will buy in stage (iii). The profit of M following this deviation will therefore be $\Pi_A^m - \pi_A(\infty, \underline{w}_B) - 2\epsilon > \Pi(\tilde{w}_A, \tilde{w}_B) - \pi_A(\infty, \tilde{w}_B) - \pi_B(\infty, \tilde{w}_A)$, so M's deviation is profitable, and we have a contradiction. A similar argument establishes that M has a profitable deviation if $\Pi_A^m - \pi_A(\infty, \underline{w}_B) < \Pi_B^m - \pi_B(\infty, \underline{w}_A)$. *Q.E.D.*

Proof of Proposition 5. Let us first consider the acceptance decisions of the downstream firms. The contract offered to R_j is such that R_j will choose to buy in stage (iii) if R_i deviates and rejects the contract offer from M in stage (ii). R_i will then get a profit of $\pi_i(\infty, \underline{w}_j)$, which is exactly what it gets in equilibrium. If R_j deviates and rejects the contract offer, R_i will still buy from M in stage (iii), and R_j will get a profit of $\pi_j(\infty, w_i^m)$, which is exactly what it gets in equilibrium. Hence, neither downstream firm has a profitable deviation.

Let us now consider possible deviations by M in stage (i). Assume that M deviates by offering contracts with $(\tilde{w}_A, \tilde{w}_B)$. If there following this deviation are to be continuation equilibria in which both downstream firms buy from M in stage (iii), R_i must be left with at least $\pi_i(\infty, \tilde{w}_j)$, and R_j must be left with at least $\pi_j(\infty, \tilde{w}_i)$. If not, they would not accept their contract offers in stage (ii). This means that the profit of M in any non-exclusionary continuation equilibrium is bounded above by $\Pi(\hat{w}_A, \hat{w}_B) - \pi_A(\hat{w}_A, \infty) - \pi_B(\hat{w}_B, \infty)$, which is weakly smaller than what M gets in the equilibrium.

Assume only one downstream firm buys from M in stage (iii) following the deviation by M. If R_A buys, the joint profit of R_A and M is bounded above by Π_A^m. R_A will never accept a contract in stage (ii) if it is left with less than $\pi_A(\infty, \underline{w}_B)$, since its flow profit by rejecting the contract offer is bounded below by this amount. The profit of M following such a deviation is therefore bounded above by $\Pi_A^m - \pi_A(\infty, \underline{w}_B)$. A similar argument establishes that the profit of M when only R_B buys following a deviation is bounded above by $\Pi_B^m - \pi_B(\infty, \underline{w}_A)$. In equilibrium, M gets $\max\{\Pi_A^m - \pi_A(\infty, \underline{w}_B), \Pi_B^m - \pi_B(\infty, \underline{w}_A)\}$, so no deviation in which only one downstream firm buys from M can be profitable. This establishes that neither the downstream firms nor M has profitable deviations. *Q.E.D.*

Proof of Proposition 6. First let us consider the acceptance decision by the downstream firms. Suppose R_i deviates and rejects M's contract offer in stage (ii). Following this deviation the rival will choose to buy at w_j^E in stage (iii) if $\pi_j(\underline{w}_j, \infty) - F_j^E > \pi_j(w_j^*, \infty) - F_j^C$, which holds by Assumption 1. This would leave R_i with a profit of $\pi_i(\infty, \underline{w}_j)$ which is exactly what it gets in equilibrium. It follows that the neither downstream firm profitably can deviate by rejecting the contract offer in stage (ii). Suppose now that R_i deviates by choosing to buy at w_i^E in stage (iii). This would give it a profit of

$-S_i + \pi_i(\underline{w}_i, w_i^*) - F_i^E$, which is exactly what it gets in equilibrium. It follows that neither downstream firm has a profitable deviation.

In this equilibrium, industry profit is maximized and the downstream firms each get the smallest conceivable profit absent restrictions on resale prices or quantities. Hence, no profitable deviations exists for M. *Q.E.D.*

Proof of Proposition 8. If one R_i deviated and rejected the offer, the rival would buy at w_i^E and R_i would be left with a profit of $\pi_i(\infty.\underline{w}_j)$ in any continuation equilibrium, which is exactly what it gets in equilibrium. A joint deviation from R_A and R_B in which both reject the offer in stage (ii) is profitable, but not self-enforcing, since R_i would have an incentive to again deviate and accept the offer from M, which would give it a profit of $\pi_i(\underline{w}_i, \infty) + \pi_i(\infty, \underline{w}_j) > \pi_i(\infty, \infty)$. *Q.E.D.*

References

Bernheim, B., B. Peleg, and M. D. Whinston (1987). Coalition-Proof Nash Equilibria I. Concepts. *Journal of Economic Theory* 42(1), pp. 1 –12.

Caprice, S. and P. Rey (2012). *Buyer power from joint listing decision.* Tech. rep. Toulouse School of Economics (TSE).

Dana Jr, J. D. (2012). Buyer groups as strategic commitments. *Games and Economic Behavior* 74(2), pp. 470–485.

Dobson, P. and M. Waterson (1999). Retailer power: recent developments and policy implications. *Economic Policy* 14(28), pp. 133–164.

Dobson, P. W. (2003). Buyer power in food retailing: the European experience. In: *OECD Conference on Changing Dimensions of the Food Economy: Exploring the Policy Issues*, pp. 6–7.

Foros, Ø. and H. J. Kind (2008). Do Slotting Allowances Harm Retail Competition? *The Scandinavian Journal of Economics* 110(2), pp. 367–384.

Hart, O. and J. Tirole (1990). Vertical integration and market foreclosure. *Brookings papers on economic activity. Microeconomics*, pp. 205–286.

Marvel, H. P. and H. Yang (2008). Group purchasing, nonlinear tariffs, and oligopoly. *International Journal of Industrial Organization* 26(5), pp. 1090–1105.

Marx, L. M. and G. Shaffer (2007). Upfront payments and exclusion in downstream markets. *The RAND Journal of Economics* 38(3), pp. 823–843.

McAfee, R. P. and M. Schwartz (1994). Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity. *The American Economic Review*, pp. 210–230.

Miklós-Thal, J., P. Rey, and T. Vergé (2011). Buyer power and intrabrand coordination. *Journal of the European Economic Association* 9(4), pp. 721–741.

Rey, P. and M. D. Whinston (2013). Does retailer power lead to exclusion? *The RAND Journal of Economics* 44(1), pp. 75–81.

Segal, I. (1999). Contracting With Externalities. *The Quarterly Journal of Economics* 114(2), pp. 337–388.