Resale price maintenance in two-sided markets*

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Abstract

We consider competing two-sided platforms selling directly to one side of the market, and through an agent to the other side. Platforms offer nonlinear tariffs, and can choose whether to contract with the same or different agents. We study the platforms' incentives to impose resale price maintenance (RPM), and the effect on end customers. We find that, even if customers on both sides value each other's participation, platforms impose minimum RPM to raise prices on both sides simultaneously if platform competition is sufficiently strong. In a linear demand example, we find that overall welfare decreases with minimum prices and increases with maximum prices.

1 Introduction

A two-sided market is characterized by a set of platforms and two distinct consumer groups, sometimes referred to as 'buyers' and 'sellers,' that value each other's participation. Many industries, particularly within 'the new economy,' fit this description; the markets for video

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game consoles, payment cards, and online auctions are typical examples of two-sided markets, and companies such as Sony, Microsoft, MasterCard, and eBay are well-known examples of platforms.

We often observe that two-sided platforms impose restrictions on the prices that can be charged by their sellers, sometimes by deciding the seller's prices directly, and other times (more indirectly) by imposing contract clauses such as resale price maintenance (RPM) or minimum advertised prices (MAP). Newspapers and magazines, for example, who sell space to advertisers and sign distribution contracts with retailers, usually impose fixed RPM on their news vendors ('cover pricing'). Online streaming platforms and marketplaces, such as Google Play Music, Spotify, and Netflix, who sell services and subscriptions to consumers, and write contracts with sellers (e.g., music publishers and studios), frequently impose restrictions on the sellers' prices, or even dictate the prices directly. Software and hardware companies (e.g., Microsoft, Apple, and Sony), who sell licenses and kits to software developers and sign contracts with retailers, often impose MAP when distributing their products (such as Windows, Xbox, iOS/Mac, iPhone, and PlayStation) to retailers.[1]

RPM is a much disputed practice in antitrust policy. Minimum or fixed RPM was until only very recently a banned practice in the U.S., through the Sherman Act; in Europe, it is still on the list of 'hardcore' infringements of competition law.[2] Policy makers have generally been much more lenient toward the use of maximum RPM, both in the U.S. and in Europe. The economic literature, however, is less clear on the subject. Several papers highlight that there may be an efficiency rationale for the use of minimum RPM, to facilitate valuable services at the retail level.[3] Another branch of the literature (presented below) shows that, in the absence of any efficiency benefits, minimum RPM, and even maximum RPM, may cause higher prices and thus harm consumers. To complicate matters further, several authors have warned that two-sided markets require a different antitrust approach from ordinary markets

[1] Sony was recently under investigation for imposing resale prices on PlayStation software for wholesalers and retailers in Japan (*Sony Computer Entertainment Inc. v. JFTC*).

[2] A case from 2007 – *Leegin Creative Leather Products, Inc. v. PSKS, Inc., 551 U.S. 877* – involved a softer treatment of minimum and fixed RPM in the U.S.

[3] See for example Telser [1960] and Mathewson and Winter [1984], as well as the quality certification argument presented by Marvel and McCafferty [1984].

(Posner [2001]; Evans [2002]; Wright [2004]).

In this paper, we analyze platforms' incentives to adopt RPM in two-sided markets. To maximize profits, a platform must take into account the indirect network externalities between its sellers and buyers, as well as the pricing strategies of rival platforms. An optimal pricing strategy will balance these concerns. However, if the platform does not directly control the prices of its sellers, an incentive problem may arise. The simplest example of this is perhaps media markets with newspapers, magazines, or TV channels that earn a high share of their profit by selling ads. Media firms may fear that their distributors charge too high prices, as this will limit the number of readers or viewers; in turn, this will cause the advertisers to reduce their participation as well.[4] To induce a lower retail price, the media firm cannot simply offer a discount to its distributor, as this also affects the firm's incentive when deciding on how much space or time to allocate to the advertisers. This therefore seems like an obvious case for imposing maximum RPM, which would be an efficient instrument for preventing high vendor prices.

To study the situation formally, we consider a model with either one or two competing platforms that sell directly to one side of the market (we refer to this as the 'direct side'), and indirectly through agents ('sellers') to the other side (the 'retail side'). Platforms may sign general nonlinear observable contracts with agents. We study a range of market configurations where we know that firms, in a corresponding one-sided market, would not benefit from the ability to impose RPM. We know for instance that a monopoly manufacturer can achieve the monopoly profit with simple nonlinear tariffs and no RPM, both when using a single agent and when using multiple competing agents. In a case with competing manufacturers, Bernheim and Whinston [1985] show that the manufacturers may induce the fully integrated profit by offering simple 'sell-out' contracts to a common agent.[5] Moreover, in a setting where the manufacturers use separate 'exclusive' agents, Bonanno and Vickers [1988] (BV hereafter)

[4] A related issue arises if platforms deal sequentially with the two sides (Hagiu [2006]) or if one side lacks information about prices on the other side (Belleflamme and Peitz [2014]; Hagiu and Halaburda [2014]).

[5] Although formally it is the manufacturers that set the final prices in Bernheim and Whinston's [1985] model, it is well known that this assumption is not important, and that the same result would arise if the agents were to control the prices instead. See for example O'Brien and Shaffer [1997] and Bernheim and Whinston [1998].

show that the firms prefer to delegate sales and pricing decisions to their agents, rather than integrating both vertically each with its own agent (which in their setting is equivalent to both manufacturers adopting RPM).

As alluded to above, adding a second direct side to these one-sided settings may change the incentives to use RPM. We therefore investigate the platforms' incentives to impose either maximum or minimum RPM on their agent(s) in a two-sided setting, and the effect of this on the final market outcome. We show that in all market configurations mentioned in the previous paragraph, but applied to a two-sided market, platforms will unilaterally prefer to adopt RPM. Sometimes the platforms will prefer to adopt maximum RPM and sometimes minimum RPM.

In our benchmark case, we show that a monopoly platform generally cannot achieve the monopoly profit with nonlinear contracts alone. However, if RPM is feasible, the platform can achieve the monopoly profit by adopting maximum RPM, both when using a single agent and multiple (differentiated) agents. Similarly, we show that two competing platforms cannot realize the fully integrated outcome with nonlinear tariffs only. When the platforms use separate agents, we find that each platform always has a unilateral incentive to adopt RPM, unlike in the corresponding one-sided setting analyzed by BV. Nevertheless, RPM is not necessarily jointly optimal for the platforms in this setting, and the equilibrium may therefore take the form of a Prisoners' Dilemma. In contrast, when using a common agent, we find that the platforms may achieve the fully integrated outcome by imposing maximum RPM if the degree of competition between the platforms is not too strong, or if the consumers on the retail side dislike participation on the opposite side (a negative indirect network effect). However, with sufficient competition, we find that the platforms may want to impose minimum prices on their common agent to prevent too low prices to consumers. Perhaps surprisingly, minimum prices are preferred by the platforms in this case specifically when each customer group welcomes more participation on the opposite side of the market (positive indirect network effects). The intuition for this is as follows. If a platform earns a high wholesale margin on each transaction with the agent, then the platform may want to charge a lower price to the buyers on the direct side of the market to boost the agent's sales. This may be the appropriate strategy if the platform already tends to set a high price on the direct side of the market, for example, if the platform is a monopolist or if the competition between platforms is relatively weak. In this situation, a maximum price is required to prevent the retail price from rising 'too high,' due to the high wholesale price. Yet, when the platforms are close substitutes, competition may drive the platforms to set prices that are 'too low' on the direct side of the market, as seen from the perspective of a fully integrated (horizontally and vertically) monopolist. To dampen competition and thus increase their prices, the platforms should therefore lower their wholesale margins. However, in turn, this may induce the agent to charge too low prices from its retail customers, again as seen from the perspective of the fully integrated firm. To prevent too low prices on the retail side, the platforms may therefore impose minimum RPM clauses. A pair of minimum prices may thus help the platforms to increase prices on both sides of the market simultaneously.

We also show that, if RPM is not feasible, the platforms may sometimes resort to using exclusive agents (different sellers) instead. The intuition is the following. For the case when the platforms would like to impose maximum prices, but are prevented from doing so, the agent will tend to set prices that are too high in equilibrium, due to double marginalization. All else being equal, by using exclusive agents, the platforms introduce some competition between the agents, and in turn this will reduce the double marginalization problem.

Our results are derived with a general demand specification. To gain a sense for the potential welfare implications, we also present a linear demand example. Here we find that, in a setting with positive indirect network effects, overall welfare increases with RPM if the platforms prefer maximum RPM, and decreases with RPM if the platforms prefer to adopt minimum RPM. Because this is just an example, we should be careful about the interpretation. Nevertheless, the general intuition for why minimum RPM (as opposed to maximum RPM) is always harmful in our setting is strong. In our framework, maximum prices are used by the platforms to reduce the margins on the retail side of the market, and this may help to increase participation on both sides of the market simultaneously. This should normally increase welfare in a two-sided market for the cases when both sides value each other's participation. Minimum prices, on the other hand, are implemented to dampen platform competition on the direct side of the market, while still facilitating high prices on the retail side. Hence, minimum RPM causes prices to increase on both sides of the market simultaneously, which will normally harm welfare in a situation where both sides value each other's participation.

Our paper extends the industrial organization literature on two-sided markets, in which Caillaud and Jullien [2003], Armstrong [2006], and Rochet and Tirole [2006] are the seminal contributions. In particular, our article is related to an emerging strand of literature that studies how two-sided platforms can make use of vertical restraints. Research in this area has shown, for example, that tying can (i) induce consumers to multi-home (Choi [2010]), (ii) soften platform competition (Amelio and Jullien [2012]), and (iii) help dominant firms to leverage market power (Choi and Jeon [2016]), and that vertical integration and exclusivity contracts can facilitate platform entry (Lee [2013]).[6] Our contribution is to consider RPM, which to our knowledge has not been done before. However, a related issue is studied by Hagiu and Lee [2011]. In a model of the video game industry, they distinguish between two business strategies: under 'outright sale,' a content provider lets his content distributor set the content price, whereas the provider retains price control under 'affiliation.' These regimes correspond to the cases with and without RPM in our paper. Hagiu and Lee [2011] show that content providers will tend to contract with one distributor exclusively whenever they give up price control. A different yet related result arises in our model, as competing platforms are more likely to enter into exclusive relations with agents (different sellers) in equilibrium, if they cannot impose RPM.[7]

Outside the field of two-sided markets, our article relates to the general literature on RPM. One strand of this literature assumes that contracts between a monopolist manufacturer and its retailers are secretly negotiated (O'Brien and Shaffer [1992]; Rey and Vergé [2004]; Montez [2015]; Gabrielsen and Johansen [2017]). These papers illustrate how either maximum or

[6] See Evans [2013] for a survey of some of this literature.

[7] Note that Hagiu and Lee [2011] label firms in a different way to us, in that our platforms correspond to their content providers, and our agents correspond to their content distributors.

minimum RPM can be used to increase final prices, by helping the monopolist overcome an opportunism problem when contracting with retailers. In addition, some papers have found that minimum RPM can be used to sustain monopoly prices in markets where both manufacturers and retailers compete (Innes and Hamilton [2009]; Rey and Vergé [2010]).[8]

Our benchmark case, with a monopoly platform, also resembles the situation studied in part of the double moral hazard literature, where a principal contracts with an agent, and where both need to make costly investments. Romano [1994], for example, considers vertical relations between a manufacturer and a retailer, and where both firms need to make investments. He finds that either maximum or minimum RPM may be used by the manufacturer to increase the industry profits, but without necessarily fully restoring the first-best outcome.

The remainder of the article is structured as follows. Section 2 outlines the formal model and presents a benchmark with a monopoly platform. Section 3 contains our main analysis of the case with competing platforms. Section 4 conducts a welfare analysis with a linear demand system. Section 5 briefly discusses some of our assumptions, and concludes. Most of the formal proofs are in the appendix.

2 The model and a benchmark

We analyze a market with either a single monopolist platform, or two similar but differentiated platforms $i \in \{1, 2\}$ that sell their products in a two-sided market. On one side of the market, each platform sells directly to its customers. We refer to this as the 'direct side,' denoted by d. On the other side of the market, we assume that the platform selects one among many homogeneous and equally efficient agents, which will resell the product on the platforms' behalf to the final customer. We refer to this as the 'retail side,' denoted by r. The platform may, for example, be a game console, like Sony's PlayStation, or a newspaper, like the New York Times, while the agent may be a traditional retailer, like Expert or Costco.[9]

[8] See also Dobson and Waterson [2007], which differs from the literature cited above by assuming that the manufacturers use (inefficient) linear wholesale prices.

[9] Note that the model may also fit for online marketplaces and streaming platforms, like Google Play Music, where the buyers on the 'direct' side are the subscribers and buyers of music tracks, while the agents on the 'retail' side are traditional sellers, like Universal Music Group. The essential feature of the setting that

Each platform incurs a constant (and symmetric) marginal cost, equal to c_d when selling to side d and c_r when selling to side r. Fixed costs are normalized to zero. The agents incur no costs except the prospective payments they make to the platform(s).

We refer to the platforms' final customers on each side of the market interchangeably as 'buyers,' 'customers,' or 'consumers.' We assume throughout that the buyers on each side pay linear prices (no fixed fees). We denote by p_s^i the price charged to the customers on side $s \in \{d, r\}$ for platform $i \in \{1, 2\}$, and we denote by $q_s^i = Q_s^i(\mathbf{p}_s, \mathbf{q}_{-s})$ the resulting quantity demanded and consumed on side s for platform i, as a function of the price(s) charged on side s, $\mathbf{p}_s = (p_s^1, p_s^2)$, and the consumption or participation on the opposite side $-s$, $\mathbf{q}_{-s} = \left(q_{-s}^1, q_{-s}^2\right)$.[10,11]

For the case of two platforms, our demand system comprises four products or services. For this system to be invertible and stable, it is required that the feedback loops between the two sides are convergent, which again holds as long as the cross-group network externalities are not too strong.[12] We therefore assume that the cross-group network externalities are 'weak' enough to ensure that our demand system has a unique solution $\widetilde{\mathbf{q}}(\mathbf{p}) = (\widetilde{q}_s^i)$ in quantities demanded as a function of the prices set on each side. In the following, we omit the tildes and denote these reduced-form quantities simply by $q_s^i(\mathbf{p}_s, \mathbf{p}_{-s})$ for $i \in \{1, 2\}$ and $s \in \{d, r\}$.

we are studying is that the participants on the direct side of the market (e.g., game developers on PlayStation or subscribers on Google Play Music) have a preference for low prices from the agent or seller (e.g., Expert or Universal Music Group), while the agent may value either high or low participation (low or high prices) on the opposite side of the market, depending on the situation. In the case of media firms, for example, we know that some consumers may dislike advertisements, in which case the readers and viewers, as well as the distributors, should have a preference for high prices to the advertisers on the direct side of the market, all else being equal.

[10] In many two-sided markets, the buyers on the two sides of the market will engage in additional 'direct' transactions (occurring at stages after the final stage of our game). Examples are advertisers who sell their goods to newspaper readers (who have seen the ads), or users of a computer operating system, who later buy software from developers that are present on the same platform. Often these transactions are exactly the reason why indirect network externalities exist on the platforms in the first place. Like much of the literature on two-sided markets, we take a reduced-form approach to these transactions. Including them in the model would add a new level of complexity, and we do not believe it would affect our qualitative results in any significant way.

[11] We do not a priori put any restrictions on whether the consumers (on both sides) participate on two platforms or only on one platform, that is, whether consumers multi-home or single-home. As long as the platforms are imperfect substitutes and consumers are heterogeneous, it is reasonable to assume that there would be a mix of multi-homing and single-homing on each side of the market.

[12] See Filistrucchi and Klein [2013] for a formal analysis. This issue applies more generally to two-sided market models with price competition at both sides.

Provided the demand system can be inverted, we make the following assumptions about the resulting reduced-form demands. First, we assume that they are continuously differentiable in all prices almost everywhere, and that the partial derivatives of q_s^i have the signs that we would expect. Specifically, we assume (i) that the goods consumed on side $s \in \{d, r\}$ are 'gross' substitutes as defined by Vives [1999, p. 145], that is, that we have both $\partial q_s^i / \partial p_s^i < 0$ and $\partial q_s^i / \partial p_s^j \geq 0$ for $i \neq j \in \{1, 2\}$, and that direct effects dominate on each side separately but also overall, that is, we have $\sum_k \partial q_s^k / \partial p_s^i < 0$ as well as $\sum_h \sum_k \partial q_h^k / \partial p_s^i < 0$, for $i \in \{1, 2\}$ and $s \in \{d, r\}$. Moreover, we assume (ii) that if the indirect network effect from side $-s$ to side s is negative ($\partial Q_s^i / \partial q_{-s}^i < 0$), then this implies $\partial q_s^i / \partial p_{-s}^i > 0$, $\partial q_s^i / \partial p_{-s}^j < 0$, and $\sum_k \partial q_s^i / \partial p_{-s}^k > 0$; and that if the indirect network effect from side $-s$ to side s is positive ($\partial Q_s^i / \partial q_{-s}^i > 0$), then this implies $\partial q_s^i / \partial p_{-s}^i < 0$, $\partial q_s^i / \partial p_{-s}^j > 0$, and $\sum_k \partial q_s^i / \partial p_{-s}^k < 0$. Note that these assumptions are closely related. For the linear demand system used in Section 4, for example, the second set of conditions is always satisfied when the products sold on side s are gross substitutes.

Throughout the analysis, we assume that buyers on side d always attach a positive value to participation on side r, that is, that $\partial Q_d^i / \partial q_r^i > 0$, and that the platform cannot be active only on the direct side, that is, if $q_r^i = 0$ then $q_d^i = 0$ as well. On the other hand, we allow buyers on side r to either value or dislike participation on side d, that is, we can have either $\partial Q_r^i / \partial q_d^i > 0$ or $\partial Q_r^i / \partial q_d^i < 0$.[13]

2.1 Timing of the game

We consider a game that proceeds in two main stages. At stage 1, each platform $i \in \{1, 2\}$ first offers a contract to one of the agents. A contract offer consists of a menu of nonlinear tariffs, with each element in the menu conditional on a specific market structure (explained below). Each nonlinear tariff $T^i(q_r^i)$ is assumed to be a function only of the quantity q_r^i of i's product distributed by the agent to side r, and (if feasible) a fixed, minimum or maximum

[13] These assumptions correspond to many real-life markets. For example, advertisers on TV, in newspapers, or in magazines always attach a positive value to consumption on the other side – without viewers/readers, the platform would not attract any advertisers at all. Readers and TV viewers, on the other hand, can be found to either like or dislike advertising, depending on the context.

resale price, v^i, which constrains the agent's price p_r^i for i's product.[14] After having observed its own contract offer(s), each agent decides whether or not to accept. Accepted contract terms are then observed by everyone. If one platform's offer is rejected at this stage, the platform is allowed to make a new public offer to a different agent, who in turn may accept or reject. If both contracts are rejected, the game ends.

We assume that the platforms make their terms $(v^i, T^i(q_r^i))$ at stage 1 conditional on whether the agent serves one or two platforms; that is, each platform i offers to the agent a menu composed of two tariffs, one tariff that applies if the agent serves both platforms i and j, and one tariff that applies if the agent only serves i. This is a natural assumption, as the optimal contract terms generally depend on whether the agent serves more than one platform. If the contract terms were not conditional on market configurations, then the platform and its agent would sometimes be stuck with an inefficient contract, in which case they would like to renegotiate. Naturally, we therefore require these conditional terms to be 'renegotiation proof,' in the sense that we will discard any contract terms that (if invoked) the platform–agent pair would prefer to publicly renegotiate before the second stage (if given the opportunity). Thus, we will not permit equilibria to be sustained by any 'implausible' off-equilibrium terms.[15]

At stage 2, each platform first decides whether or not to be active. If platform i decides not to be active, then we simply have $q_r^i = q_d^i = 0$ in the continuation game. Next, all active firms set prices, which are observed by all customers before demand is realized.[16] Finally, payments are completed according to the terms of trade accepted at stage 1.

Given our assumptions, we can write the profit of any agent a as $\pi^a = \sum_{k\in\{1,2\}}\{p_r^k q_r^k - T^k\left(q_r^k\right)\}$ if it sells the goods of both platforms, and simply $\pi_i^a = p_r^i q_r^i - T^i(q_r^i)$ if it sells the goods of platform i only. Similarly, platform i's profit can be written as $\pi^i = (p_d^i - c_d)\, q_d^i + T^i(q_r^i) - c_r q_r^i$. The tariff $T^i(q_r^i)$ can take a wide variety of forms but is assumed to be

[14] In principle, one could also allow the tariff to depend on the price and quantity on side d. We assume that this is not possible. See the discussion in Section 5.

[15] By 'off-equilibrium terms' we mean terms that appear in the menu but are not invoked by any agent in equilibrium. We require these terms to be immune to public renegotiation should they be accepted.

[16] Our qualitative result extends also to the case in which the platforms compete in quantities on the direct side of the market. The analysis of quantity competition was part of an earlier draft of the paper.

differentiable almost everywhere. For the case when the accepted contracts are of the type $T^i(q_r^i) = F^i + w^i q_r^i$ for $i \in \{1,2\}$, we assume that a unique Nash equilibrium $\mathbf{p}^*(\mathbf{w})$ exists at the final stage of the game, as in the linear demand example presented in Section 4.

We continue to solve the game in the usual way, looking for a subgame-perfect equilibrium $(\mathbf{v}^*, \mathbf{T}^*, \mathbf{p}^*)$. We focus on symmetric equilibria that are Pareto undominated for the platforms.

2.2 Benchmark: A monopoly platform

We start by analyzing the benchmark situation with a monopoly platform. Because we are dealing with only one platform, we simply drop the superscripts i and j for the remainder of this section.

We first consider the fully integrated (industry profit-maximizing) outcome, that is, the situation in which the platform is able to sell directly to both sides of the market simultaneously. Industry profits are $\Pi = \sum_{s\in\{d,r\}} (p_s - c_s) q_s$, and reach a maximum at some price vector that we denote by $\mathbf{p}^M = \left(p_d^M, p_r^M\right)$. We let $\Pi^M = \Pi\left(\mathbf{p}^M\right)$ denote industry profits when prices are set equal to $\mathbf{p}^M$.

The fully integrated firm's first-order conditions, evaluated at $\mathbf{p}^M$, are then given by

$$q_d^M + \sum_{s\in\{d,r\}} \left(p_s^M - c_s\right) \left.\frac{\partial q_s}{\partial p_d}\right|_{\mathbf{p}^M} = 0, \tag{1}$$

for the price on side d, p_d, and analogously,

$$q_r^M + \sum_{s\in\{d,r\}} \left(p_s^M - c_s\right) \left.\frac{\partial q_s}{\partial p_r}\right|_{\mathbf{p}^M} = 0, \tag{2}$$

for the price on side r, p_r. Here, q_d^M and q_r^M represent the participation on each side when prices are equal to $\mathbf{p}^M$. We assume that the monopoly markup on each side can take any value, positive or negative, $p_s^M - c_s \lesseqgtr 0$, for $s \in \{d,r\}$.[17]

[17]Obviously, they cannot both be negative at the same time. Yet, we know that platforms sometimes incur a loss on one side of the market while making a profit on the other side. It is well known that Sony, for example, often makes a loss when selling video game consoles to their final customers, while they make up for it by charging their video game developers. In the same way, many newspapers make a loss when

Now suppose that the platform must use an agent to sell to side r. First, we consider the case when RPM is not allowed. At stage 1, the platform offers a tariff T to one of the agents. Suppose the agent accepts the offer. Given the contract T, and given that the platform decides to be active, the platform sets its price to side d while the agent sets the price to side r. We let $\left(p_r^B, p_d^B\right)$ be the equilibrium prices at stage 2 as functions of the accepted contract terms Tthat the equilibrium contract terms $T\left(q_r\right)$ will have to maximize:

$$\left(p_d^B - c_d\right) q_d\left(p_d^B, p_r^B\right) - c_r q_r\left(p_r^B, p_d^B\right) + T\left(q_r\right) \tag{3}$$

$$\text{subject to } p_r^B q_r\left(p_r^B, p_d^B\right) - T\left(q_r\right) = 0.$$

We can then state the following lemma.

Lemma 1 *If (T^*, p_r^*, p_d^*) forms a subgame-perfect equilibrium, then the tariff T^* is continuous and differentiable at the quantity q_r^* induced by (p_r^*, p_d^*).*

Proof. See Appendix A.

In addition to simplifying the rest of the analysis, Lemma 1 also provides valuable insights into which contract arrangements do not occur in any equilibrium. Lemma 1 states that, in equilibrium, a slight increase or decrease in the quantity q_r sold to side r cannot induce a discontinuous change in the payment from the agent to the platform. The accepted tariff may have discontinuities, but the point of discontinuity (the threshold value) of, say, q_r cannot be equal to the equilibrium quantity $q_r^* = q_r\left(\mathbf{p}^*\right)$, because then $\mathbf{p}^*$ would not be immune to profitable deviations.

The intuition for this is straightforward. In a two-sided market, the quantity sold to side r is a function of the quantity sold to the buyers on the direct side of the market. Hence, if we were to marginally move away from the equilibrium quantity q_r^*, in either direction, and T^* were to 'jump up,' then the platform could induce a discrete increase in the agent's payment

distributing their papers to readers (e.g., we see many free newspapers) while they make up for it by charging their advertisers.

(and thus a discrete increase in its own profit) by marginally adjusting its price p_d to side d either up or down, so as to cause a slight increase or decrease in the quantity sold to side r. Obviously, the payment T^* cannot 'jump down' either, otherwise the agent could induce a discrete reduction in its payment to the platform (and thus a discrete increase in its own profit) by marginally adjusting its price p_r to side r either up or down.[18] Note that this result gives us a reason to focus on price restraints, as we do in the following, and not, for example, on quantity restraints or sales-forcing contracts.[19]

Given that the contract T is accepted, the firms proceed to set their prices at stage 2. We note that the platform and the agent will not be able to achieve the fully integrated outcome, even if they are both monopolists. To see this, note that Lemma 1 and our assumptions on demand imply that the agent's and the platform's profit functions are differentiable at the equilibrium point. The agent's first-order condition for profit maximization is therefore

$$(p_r - T') \frac{\partial q_r}{\partial p_r} + q_r = 0, \tag{4}$$

while the platform's first-order condition is

$$(p_d - c_d) \frac{\partial q_d}{\partial p_d} + (T' - c_r) \frac{\partial q_r}{\partial p_d} + q_d = 0. \tag{5}$$

For $\left(p_d^M, p_r^M\right)$ to form an equilibrium, (2) and (4) have to be aligned when evaluated at the optimal prices. By using the implicit function theorem, we obtain the condition

$$T' - c_r = -\left(p_d^M - c_d\right) \frac{\partial Q_d}{\partial q_r} \lesseqgtr 0, \tag{6}$$

which states that the platform's markup on the retail side should be negative when $p_d^M - c_d > 0$ for the agent to fully take into account the positive feedback on the platform's sales on the direct side, or positive when $p_d^M - c_d < 0$, to take into account the platform's loss on the

[18] Note that the intuition for this result resembles the intuition for Lemma 1 in O'Brien and Shaffer [1992].

[19] We briefly discuss these and alternative vertical restraints in Section 5.

direct side.[20] On the other hand, it is easy to see that to induce the optimal price to side d, the platform needs to earn the full monopoly rent on the last unit sold to its agent. That is, for the platform's incentives to be aligned with the fully integrated firm, we require that $T' - c_r = p_r^M - c_r$ when $q_r = q_r^M$, which is generally incompatible with condition (6). This gives us the following result.

Proposition 1 *(Monopoly platform)*
In a subgame-perfect equilibrium, a monopoly platform

- *cannot induce the fully integrated outcome Π^M with only a nonlinear tariff $T(q_r)$;*
- *can, if RPM is feasible, induce the fully integrated outcome Π^M by imposing a fixed or maximum resale price $v = p_r^M$, and squeezing the agent's margin on the last unit sold on the retail side ($T' = p_r^M$).*

An important insight from the literature on vertical restraints is that a successive monopoly (and also common agency settings with competing suppliers) can achieve the first-best level of profit by using simple nonlinear contracts, for example, a two-part tariff with a marginal wholesale price equal to the manufacturer's marginal cost. Such a sell-out contract will avoid double marginalization and allow the agent to maximize industry profits. The monopoly profit can then be shared or collected through a positive fixed fee. The first part of Proposition 1 shows that in a two-sided market this does not work. The reason is that the marginal wholesale price needs to be set high enough for the platform to fully take into account the indirect network effects when setting the price on side d; a high marginal wholesale price will cause the agent to set the price on side r too high. The second-best contract (without RPM) therefore involves setting the marginal wholesale price below the monopoly price to side r, yet above the level that would secure the monopoly price to side r. All else being equal, this will cause too few sales to the retail side of the market, and either too many (in the case of a negative indirect network effect $\partial Q_r/\partial q_d < 0$) or too few (in the case of a positive indirect

[20] We can use the implicit function theorem here because our demand system is invertible under the assumption that cross-group externalities are not too strong.

network effect $\partial Q_r/\partial q_d > 0$) sales to the direct side of the market. The platform is therefore left unable to extract its full monopoly profit. The second part of Proposition 1 shows that this problem can be fully solved by RPM.

Note that the platform always imposes (a fixed or) maximum price at stage 1, irrespective of the sign of the monopoly markup $p_s^M - c_s$ on each side $s \in \{d, r\}$. A maximum resale price commits the agent not to set the price too high on the retail side. This implies that the platform is free to set the marginal wholesale price on the last unit sold equal to p_r^M at stage 1, which in turn will induce the platform to set the correct price p_d^M on the direct side of the market at stage 2. Without a maximum price, and given of course that q_r^M is strictly positive, a marginal wholesale price equal to p_r^M will induce the agent to charge a price above the fully integrated monopoly price, $p_r > p_r^M$. Thus, a minimum price is never appropriate in the case of a monopoly platform.

In our linear demand example in Section 4 below, we find that, in the typical case with positive indirect network effects, maintained prices (weakly) increase the overall surplus for both the platforms and the buyers when platforms do not compete. The intuition for this is straightforward. Given that RPM is not feasible, the platform will face a double marginalization problem on the retail side of the market, which requires the platform to reduce the marginal wholesale price below p_r^M. In reducing the wholesale price, however, the platform becomes inclined to set a higher price on the direct side of the market, because it will partially ignore the positive indirect network effect on the retail side. In balancing these two concerns, the wholesale price therefore ends up too high to induce the correct price on the retail side, and too low to induce the correct price on the direct side. Hence, the prices end up above the fully integrated level on both sides of the market simultaneously, in a situation where the consumers on both sides would prefer lower prices overall. In this case, a maximum price clearly would lead to an increase in overall welfare.

The welfare effect becomes ambiguous, however, once we consider the possibility for a negative indirect network effect on the retail side of the market, $\partial Q_r/\partial q_d < 0$. In this case, a maximum price may cause a reduction in the price on the retail side, as before, but then it may cause the price to increase on the direct side. The reason is the following. Without RPM, the platform will set the marginal wholesale price below p_r^M to combat double marginalization on the retail side. Yet, the lower wholesale price will cause the platform to partially ignore the negative indirect network effect on the retail side, which in turn implies that it will set the price too low on the direct side. Hence, in balancing the two effects, the price may become too high on the retail side, and too low on the direct side, as seen from the perspective of the fully integrated firm. In this case, a maximum price will benefit the platform as well as the platform's customers on the retail side, while it may reduce demand and therefore possibly harm the buyers on the direct side. This case is made more complicated by the fact that the typical example of negative network effects in two-sided markets is advertising, which at least in some cases may be considered wasteful.[21]

Note that the analysis presented above is an example of a more general phenomenon, in which a manufacturer, after having agreed on terms of trade with his retail agent, has to make decisions about additional activities that may enhance the demand for his product (such as services and advertising).[22] In this case, we know that a maximum RPM clause will tend to solve the problem for the manufacturer. The differences in our case of a two-sided market are that (i) the fully integrated monopolist is making revenues on both sides of the market simultaneously, and it could be making a loss on the retail side (unlike a one-sided market, where all profits are generated on the retail side), and that (ii) the feedback effects are going both ways (positive or negative) because of the two buyer groups. Despite these differences, however, the analysis so far reveals that (a fixed or) maximum RPM is always the appropriate price restraint for a monopoly platform, just as for a monopoly manufacturer making noncontractible investments. The reason, as explained above, is that the monopoly platform always wants to squeeze the agent's margin by setting the marginal wholesale price on the last unit equal to p_r^M, such that (a fixed or) maximum RPM is needed to prevent a too high price on side r. In the next section, we show how this conclusion may change once

[21] In the case of purely informative (and nonwasteful) advertising, we should perhaps expect the indirect network effect to be positive on the retail side, because advertising then helps to improve the matching of consumers and products.

[22] See, for example, Romano [1994].

we introduce platform competition. Note finally that the insight from Proposition 1 extends to settings in which the platform uses multiple competing agents to distribute its product on the retail side. As long as each agent has some degree of market power downstream, they will set their prices above the marginal wholesale price. This means that the platform's marginal wholesale price needs to be set below p_r^M on the final unit to induce the monopoly price on side r. In turn, this implies that the platform is not induced to set the monopoly price p_d^M to side d. The platform may again solve the problem by imposing a maximum resale price on its agents.[23,24]

3 Competing platforms

In a setting where a two-sided monopoly platform distributes its product to one side of the market via an intermediary, our benchmark above shows that there is a rationale for the platform always to adopt a maximum (or fixed) RPM clause. This result also suggests that maintained prices sometimes may be good for the platform's customers, as it may allow the monopolist to internalize all the indirect network externalities and reduce prices. This reasoning does not necessarily apply to the case of competing platforms, as we will now see.

When we turn to the case of competing platforms, we may first take advantage of the fact that Lemma 1 easily extends to the case of competing platforms.[25]

Lemma 2 *If* $(\mathbf{T}^*, \mathbf{p}^*)$ *forms a subgame-perfect equilibrium in the game with competing platforms, then for each platform* $i \in \{1, 2\}$ *the accepted tariff* T^{i*} *is continuous and differentiable at the quantity* q_r^{i*} *induced by* $\mathbf{p}^*$.

[23] Note that the problem becomes less severe the fiercer is the competition between downstream agents. At the limit, when the agents are perfect substitutes, the agents' downstream margins are zero, and hence the platform may achieve the first-best outcome without RPM.

[24] An interesting extension would be to look at the case in which the agents' contracts with the two-sided monopolist are unobservable to rival agents. O'Brien and Shaffer [1992] show that a one-sided monopolist (who does not engage in advertising activities) will behave opportunistically and set the wholesale prices too low if contracts are unobservable. They also show that maximum RPM may be used to restore the fully integrated outcome in this setting. In a recent paper, Gabrielsen and Johansen [2017] show that this RPM equilibrium breaks down if the retail agents also offer services to customers that increase the overall demand for the monopolist's product. It would be interesting to see if something similar happens in a setting without any services or promotional activities, but with a two-sided monopolist.

[25] This holds for any subgame, that is, whether the platforms use a common agent or exclusive agents.

Proof. See Appendix A.

The following is an example of a relatively simple nonlinear tariff for platform $i \in \{1,2\}$ that satisfies Lemma 2 and that also proves to be sufficiently general for the platforms.

$$T^i\left(q_r^i\right) = \begin{cases} w^i q_r^i + F^i & \text{if } \; q_r^i > 0 \\ f^i & \text{if } \; q_r^i = 0 \end{cases} \tag{7}$$

Here, w^i is a (constant) marginal wholesale price, and $F^i \lesseqqgtr 0$ and $f^i \lesseqqgtr 0$ are 'fixed' transfers.[26] To simplify the rest of the exposition, we proceed by assuming that each platform uses a tariff of this form. Note that if either $f^i = 0$ or $f^i = F^i$, this amounts to a typical two-part tariff.

We again start with the fully integrated (industry profit-maximizing) outcome as a benchmark, that is, the situation with a single firm that is fully integrated both horizontally and vertically. The overall profit is now given by $\Pi(\mathbf{p}) = \sum_{s\in\{d,r\}} \sum_{k\in\{1,2\}} \left(p_s^k - c_s\right) q_s^k$, and we again assume that it reaches its maximum for some unique price vector, which we denote by $\mathbf{p}^M = \left(p_d^M, p_d^M, p_r^M, p_r^M\right)$. In the same way as before, we denote by $\Pi^M = \Pi\left(\mathbf{p}^M\right)$ the industry profits when the prices are set equal to $\mathbf{p}^M$, and by $q_s^M = q_s^i\left(\mathbf{p}^M\right)$ the quantities.

The fully integrated firm's first-order conditions, evaluated at the optimal prices $\mathbf{p}^M$, are now equal to

$$q_r^M + \sum_{s\in\{d,r\}} \sum_{k\in\{1,2\}} \left(p_s^M - c_s\right) \left.\frac{\partial q_s^k}{\partial p_r^i}\right|_{\mathbf{p}^M} = 0, \tag{8}$$

for the price on side r, and

$$q_d^M + \sum_{s\in\{d,r\}} \sum_{k\in\{1,2\}} \left(p_s^M - c_s\right) \left.\frac{\partial q_s^k}{\partial p_d^i}\right|_{\mathbf{p}^M} = 0, \tag{9}$$

for the price on side d, for $i \in \{1,2\}$. To keep the analysis tractable, we assume henceforth that the fully integrated monopoly markups are nonnegative on the direct side, $p_d^M - c_d \geq 0$, while they can take any value on the retail side, $p_r^M - c_r \lesseqqgtr 0$. We cover the case $p_d^M - c_d < 0$ in Appendix B.

[26] Note that this does not amount to an extension of the contract space compared with what we allowed in the monopoly benchmark.

Now, suppose instead that the platforms are competing and that they must use retail agents to sell their goods to side r. In the case of competing platforms, one of two situations may arise, depending on whether the platforms make their offers to a common agent or to different 'exclusive' agents at stage 1 of the game.

3.1 Competing vertical structures

Suppose first that the platforms offer contracts to separate exclusive agents, that is, we have a case with 'competing vertical structures.' Alternatively, suppose that the platforms make offers to the same agent, but that the agent rejects one of the offers, in which case the rejected platform makes a new offer to a different agent.[27] Given that the contracts are renegotiation proof, these two scenarios are equivalent.

Since the influential paper by BV, it has been a well-known insight from the one-sided literature that rival firms may collectively benefit from delegating their pricing decisions to independent retail agents. Such arm's length contracting allows the firms to precommit to higher final prices, by imposing marginal wholesale prices in excess of marginal costs. Given that prices are strategic complements, this creates an overall lessening of competition in the final market, and the resulting increase in profits may be redistributed through fixed fees or equivalent.

Even though the original BV paper was concerned with vertical integration versus separation, it is obvious that there would be no strategic advantage of vertical separation in their setting if it did not also come with sufficient discretion for the retailer when setting the final price. It is worth noting, however, that in the BV equilibrium, given the strategy of the rival to delegate the pricing decision, at the contracting stage, each manufacturer is actually indifferent between (i) charging a marginal wholesale price above the marginal cost, and (ii) imposing a high final price directly through RPM (if feasible). Both strategies may achieve the same commitment to a high final price, and thus the delegation equilibrium is a weak Nash

[27] Note that it is thus impossible to fully exclude a rival platform from the market, as the rival could just make a new offer after being rejected once. The only reason for a platform to induce its agent to reject the rival is therefore to force a subgame with competing vertical structures.

equilibrium, if one considers RPM to be a feasible alternative for the firms.[28] In a two-sided market, on the other hand, the platforms are not indifferent if put in the same situation, and hence they will always adopt RPM in equilibrium, if allowed to, which we demonstrate in the following.

To see that adoption of RPM is a strictly dominant strategy for the platforms, suppose first that platform j only uses a two-part tariff with terms (w^j, F^j) and no RPM. Let $P_d^{Bj}(p_d^i, p_r^i; w^j)$ and $P_r^{Bj}(p_r^i, p_d^i; w^j)$ be the best-response functions of platform j and its agent, respectively, at the final stage, that is, the prices that simultaneously maximize $\pi^j = \left(p_d^j - c_d\right) q_d^j + (w^j - c_r)\, q_r^j + F^j$ with respect to p_d^j, and $\pi_j^a = (p_r^j - w^j)\, q_r^j - F^j$ with respect to p_r^j, while holding (p_r^i, p_d^i) fixed. We may then write the first-stage joint profit of platform i and its agent as a function of their second-stage prices (p_d^i, p_r^i) directly.

$$\begin{aligned}\tilde{\Pi}^i\left(p_r^i, p_d^i\right) &= \left(p_r^i - c_r\right) q_r^i\left(p_r^i, P_r^{Bj}, p_d^i, P_d^{Bj}\right) \\ &\quad + \left(p_d^i - c_d\right) q_d^i\left(p_d^i, P_d^{Bj}, p_r^i, P_r^{Bj}\right)\end{aligned} \tag{10}$$

As in the monopoly case, we know that in any candidate equilibrium, the contract offered by platform i to its agent has to maximize the platform's profit, subject to the agent's zero-profit condition, which implies that the two are maximizing their joint profit, taking as given the contract terms signed between platform j and its agent. Maximization of the joint profit of platform i and its agent requires that both $\partial\tilde{\Pi}^i/\partial p_r^i = 0$ and $\partial\tilde{\Pi}^i/\partial p_d^i = 0$. Let $(\tilde{p}_r^i, \tilde{p}_d^i)$ be the prices that simultaneously solve these first-order conditions. We then note that, without RPM, the second-stage first-order conditions for platform i and its agent, respectively, are

$$\left(w^i - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} + \left(p_d^i - c_d\right)\frac{\partial q_d^i}{\partial p_d^i} + q_d^i = 0 \tag{11}$$

[28] If one or both firms deviate and use RPM, then the BV delegation equilibrium can no longer be sustained. Hence, if RPM is feasible, at least two types of equilibria may arise: one in which neither firm adopts RPM at the contracting stage (the BV delegation equilibrium), and a second in which both firms adopt (fixed or maximum) RPM. The latter type gives the same outcome as the equilibrium with vertical integration.

with respect to p_d^i, and

$$\left(p_r^i - w^i\right)\frac{\partial q_r^i}{\partial p_r^i} + q_r^i = 0 \tag{12}$$

with respect to p_r^i. Thus, we can show that for the first-stage and second-stage incentives to be aligned with respect to p_d^i, we need that

$$w^i - c_r = \left(p_r^i - c_r\right)\frac{\frac{\partial q_r^i}{\partial p_d^i} + \sum_{s\in\{r,d\}}\frac{\partial P_s^{Bj}}{\partial p_d^i}\frac{\partial q_r^i}{\partial p_s^j}}{\frac{\partial q_r^i}{\partial p_d^i}} + \left(p_d^i - c_d\right)\frac{\sum_{s\in\{r,d\}}\frac{\partial P_s^{Bj}}{\partial p_d^i}\frac{\partial q_d^i}{\partial p_s^j}}{\frac{\partial q_r^i}{\partial p_d^i}}, \tag{13}$$

and similarly, for the first-stage and second-stage incentives to be aligned with respect to p_r^i, we need that

$$w^i - c_r = \left(p_r^i - c_r\right)\frac{\sum_{s\in\{r,d\}}\frac{\partial P_s^{Bj}}{\partial p_r^i}\frac{\partial q_r^i}{\partial p_s^j}}{-\frac{\partial q_r^i}{\partial p_r^i}} + \left(p_d^i - c_d\right)\frac{\frac{\partial q_d^i}{\partial p_r^i} + \sum_{s\in\{r,d\}}\frac{\partial P_s^{Bj}}{\partial p_r^i}\frac{\partial q_d^i}{\partial p_s^j}}{-\frac{\partial q_r^i}{\partial p_r^i}}, \tag{14}$$

with both conditions evaluated at prices $(\tilde{p}_r^i, \tilde{p}_d^i)$ for platform i. We observe that conditions (13) and (14) are generally not the same, and hence the pricing incentives at the first and second stages are generally not aligned. Platform i would therefore strictly prefer (i) to impose RPM on its agent at the first stage of the game, setting $v^i = \tilde{p}_r^i$, and (ii) to set the marginal wholesale price w^i in accordance with (13) when evaluated at $(\tilde{p}_r^i, \tilde{p}_d^i)$. Together this will ensure that platform i and the agent set their prices equal to $(\tilde{p}_r^i, \tilde{p}_d^i)$ at the final stage of the game.

It is easy to check that a similar deviation to RPM would benefit the platform also when the rival platform imposes RPM. The differences are that, in conditions (13) and (14), platform j's second-stage best-response function P_d^{Bj} is now a function of the first-stage price v^j that platform j imposes on its agent, and that $\partial P_r^{Bj}/\partial p_r^i = \partial P_r^{Bj}/\partial p_d^i = 0$ (given of course that v^j is binding in equilibrium). Again, the first-stage and second-stage incentives are not perfectly aligned without the use of RPM by platform i, and thus we may conclude that it is a strictly dominant strategy for each platform to impose RPM at the first stage of the game.

Proposition 2 *(Competing vertical structures)*
In the subgame in which the platforms use exclusive agents, it is a strictly dominant strategy for each platform to impose a binding RPM clause, if feasible. Whether the equilibrium RPM clause can be a minimum or maximum clause (or fixed) will depend on (i) the degrees of substitution between the platforms, (ii) the sizes and signs of the indirect network effects, and (iii) the marginal production cost on each side of the market.

In the following, we let Π_N^{**} and Π_N^{*} denote the equilibrium industry profits in the subgame with competing vertical structures, with and without RPM, respectively. Obviously, both profits Π_N^{**} and Π_N^{*} are strictly below the industry profit maximum Π^M, as long as the platforms (and their agents) are competing. Under certain conditions in a two-sided market, price delegation may collectively benefit the two platforms, just as it would in a one-sided market (BV); that is, we may have $\Pi_N^{**} < \Pi_N^{*}$. However, Proposition 2 states that, different from a one-sided market, price delegation is never an equilibrium strategy as long as RPM is feasible. Thus, the adoption of RPM turns out to be a Prisoners' Dilemma for the two platforms in some situations ($\Pi_N^{**} < \Pi_N^{*}$), while they collectively benefit from RPM in other situations ($\Pi_N^{**} > \Pi_N^{*}$).[29]

3.2 Common agency

Suppose next that the platforms choose a common agent at the first stage of the game. Before we move on, it is useful to note that a pair of nonlinear tariffs of the type (7) is now sufficient to induce each platform to offer contract terms at the first stage of the game that attempt to maximize the overall joint profit created at the second stage. More formally, we have that in any subgame-perfect common agency equilibrium that is Pareto undominated for the

[29] We are able to show by examples that both situations, $\Pi_N^{**} < \Pi_N^{*}$ and $\Pi_N^{**} > \Pi_N^{*}$, can occur.

platforms, the contract terms $(\mathbf{v}, \mathbf{w}, \mathbf{F})$ will have to maximize

$$\sum_{i\in\{1,2\}} \left\{ \left(p_d^{Bi}\left(w^i, w^j\right) - c_d\right) q_d^i\left(\mathbf{p}^B\right) + \left(w^i - c_r\right) q_r^i\left(\mathbf{p}^B\right) + F^i \right\} \tag{15}$$

$$\text{subject to } \sum_{i\in\{1,2\}} \left\{ \left(p_r^{Bi}\left(w^i, w^j\right) - w_i\right) q_r^i\left(\mathbf{p}^B\right) - F^i \right\} = 0,$$

where $\mathbf{p}^B = \left(p_s^{Bi}\left(w^i, w^j\right)\right)$ is the vector of equilibrium prices at stage 2, as functions of the contract terms signed at stage 1.[30] As an example, suppose RPM is not feasible. What the claim above implies, is that in any Pareto undominated common agency equilibrium, each platform $i \in \{1, 2\}$ is induced to offer to the agent the marginal wholesale price w^i that maximizes the function $\Pi\left(\mathbf{p}^B\left(w^i, w^j\right)\right)$, taking the rival's marginal wholesale price w^j as given.[31]

However, just as in the case with a monopoly platform, a pair of nonlinear tariffs will not be sufficient to induce the overall first-best outcome Π^M for the platforms and the agent. To see this, note that, when both platforms are active, the agent's first-order condition with respect to its retail price p_r^i is

$$q_r^i + \sum_{k\in\{1,2\}} \left(p_r^k - w^k\right) \frac{\partial q_r^k}{\partial p_r^i} = 0 \tag{16}$$

for $i \in \{1, 2\}$, while for the platform i, the first-order condition is

$$q_d^i + \left(p_d^i - c_d\right) \frac{\partial q_d^i}{\partial p_d^i} + \left(w^i - c_r\right) \frac{\partial q_r^i}{\partial p_d^i} = 0. \tag{17}$$

To induce the fully integrated prices on side r, we know that (8) and (16) have to be aligned. Evaluating the two expressions at the prices $\mathbf{p}^M$, and using the implicit function

[30] This includes both the situation when RPM is feasible and when it is not. In the former case, we simply have $p_r^{Bi}\left(w^i, w^j\right) = v^i$ for $i \neq j \in \{1, 2\}$, given of course that the RPM clauses are binding in equilibrium. If RPM is not feasible, then we simply have that v^i is not binding in equilibrium.

[31] This claim is proved in Appendix A, in the proofs of Proposition 3 and 4 below.

theorem, we obtain the condition

$$w^i - c_r = -\left(p_d^M - c_d\right) \sum_{k\in\{1,2\}} \left.\frac{\partial Q_d^k}{\partial q_r^i}\right|_{\mathbf{p}^M} \leq 0. \tag{18}$$

On the other hand, to induce the fully integrated prices on side d, (9) and (17) have to be aligned. When evaluating the two expressions at the optimal prices $\mathbf{p}^M$, we obtain the condition

$$w^i - c_r = \left(p_r^M - c_r\right) + \left.\frac{\sum_{s\in\{d,r\}} \left(p_s^M - c_s\right) \frac{\partial q_s^j}{\partial p_d^i}}{\frac{\partial q_r^i}{\partial p_d^i}}\right|_{\mathbf{p}^M} \lesseqgtr 0. \tag{19}$$

(18) and (19) are generally not the same. Hence, we can conclude that it is not possible to induce the fully integrated outcome with only nonlinear tariffs.[32]

Define $\mathbf{p}_C^* = (p_{C,r}^*, p_{C,r}^*, p_{C,d}^*, p_{C,d}^*)$ as the prices in a common agency situation that we obtain by (i) simultaneously solving all the firms' first-order conditions (16) and (17) at stage 2, and then (ii) solving the maximization problem (15) at stage 1. Define as Π_C^* the industry profit when prices are equal to $\mathbf{p}_C^*$. We can then state the following result.

Proposition 3 *(Common agency – no RPM)*

If RPM is not feasible, then a nonlinear tariff $T^i\left(q_r^i\right)$ for each platform $i \in \{1,2\}$ is generally not sufficient to induce the fully integrated outcome Π^M. As a consequence, a common agency equilibrium is not guaranteed to exist.

- *If $\Pi_C^* \geq \Pi_N^*$, then a Pareto undominated subgame-perfect common agency equilibrium exists in which each platform collects the profit $\Pi_C^*/2$ at stage 2.*

- *If $\Pi_C^* < \Pi_N^*$, then a subgame-perfect common agency equilibrium does not exist.*

Proof. See Appendix A.

[32] A similar insight appears in an earlier version of Kind *et al.* [2016], who use a linear demand model to analyze a TV industry with viewers who dislike advertising.

Proposition 3 extends the first part of Proposition 1 to the case of competing platforms. To induce the agent to take into account the positive indirect network effects exerted on the direct side of the market, the platforms should sell their goods at a wholesale price at or below cost according to (18). The intuition is clear. Starting from a wholesale price at which the agent fully internalizes the platform's production cost ($w^i = c_r$), the platform could gain by further incentivizing the agent's sales on the retail side, as these sales will boost demand and possibly profits (given that $p_d^M - c_d > 0$) on the direct side. On the other hand, the wholesale margin should also take into account the platform's incentives when selling to the direct side of the market, which (because of platform competition) now includes the incentive to set low prices to steal customers from the rival. The latter implies that the appropriate wholesale margin (19) could be either positive or negative, depending on the indirect network externalities and the degree of competition between the platforms. Obviously, the wholesale margin cannot (on a general basis) achieve both goals simultaneously, and the platforms are therefore unable to extract monopoly rents when using a common agent and not using RPM.

To see that a common agency equilibrium always exists whenever $\Pi_C^* \geq \Pi_N^*$, suppose that each platform $i \in \{1, 2\}$ offers to the agent contract terms (w_C^*, F_C^*, f_C^*), contingent on the agent accepting both offers, and (w_N^*, F_N^*, f_N^*), contingent on the agent accepting platform i's offer only. Because the terms (w_N^*, F_N^*, f_N^*) are renegotiation proof, we know that they will have to induce the profit $\Pi_N^*/2$ for each platform whenever an offer is rejected by the agent. To sustain a common agency equilibrium when $\Pi_C^* \geq \Pi_N^*$, we therefore need that each platform earns at least $\Pi_C^*/2$ when both offers are accepted. Suppose, therefore, first that F_C^* is set according to

$$\left(p_{C,d}^* - c_d\right) q_{C,d}^* + \left(w_C^* - c_r\right) q_{C,r}^* + F_C^* = \frac{\Pi_C^*}{2}, \tag{20}$$

where $q_{C,s}^* = q_s^i\left(\mathbf{p}_C^*\right)$ for $s \in \{d, r\}$ and $i \in \{d, r\}$, and second, that $f_C^* = \Pi_C^*/2$. These terms will ensure zero profit to the agent and $\Pi_C^*/2$ to each platform in the second-stage equilibrium, and thus will be accepted by the agent if offered by both platforms. Moreover, the terms also ensure platform i a profit of at least $\Pi_C^*/2$, even if the agent accepts different terms from platform j.[33] To see this, suppose platform j offers a marginal wholesale price different from w_C^*. Two things can happen if the agent accepts the new wholesale price: either the deviation causes the second-stage profit of platform i to increase above $\Pi_C^*/2$ whenever $q_r^i > 0$ (in which case, by definition, the profit for the agent and platform j will have to decrease), or the second-stage profit of platform i falls below $\Pi_C^*/2$ whenever $q_r^i > 0$, in which case platform i will opt not to trade with the agent at the second stage ($q_r^i = 0$), and it will collect the fee $f_C^* = \Pi_C^*/2$ instead (in which case the profit of the agent and platform j again will have to decrease). Thus, there is no way for the agent and a platform to deviate from the proposed equilibrium, without either (i) hurting its own profit or (ii) inducing the agent to reject at least one offer (in which case we are back in the subgame with competing vertical structures). Hence, we can conclude from this that a common agency equilibrium exists as long as $\Pi_C^* \geq \Pi_N^*$.[34]

Different from the monopoly case (Proposition 1), and also different from a one-sided market, if the platforms were to adopt a pair of 'standard' two-part tariffs (i.e., with $f^i = 0$ or $f^i = F^i$), then profitable deviations would exist and the outcome Π_C^* could not be sustained. The core of the problem is that, in a two-sided market, the platforms earn 'upstream' margins that are not zero whenever they are active at the second stage. With two-part tariffs, platform i will typically continue to be active following a marginal deviation by platform j away from w_C^*, and platform j therefore fails to take into account the full impact on platform i's profit when deviating together with the agent (simply because platform i's 'upstream' margins are not part of j's joint profit with the agent), and vice versa.

Note finally that, unlike a one-sided market, because the platforms are unable to achieve the fully integrated profit when using a common agent, this opens up the possibility that the platforms sometimes will use exclusive agents instead whenever RPM is not feasible. The intuition for this is as follows. In a two-sided market, according to (18), the wholesale price

[33] Note that the tariffs analyzed here are structurally similar to (and have the same strategic value as) the nonlinear tariffs analyzed by Miklós-Thal *et al.* [2011] and Rey and Whinston [2013], both based on the upfront payments introduced in Marx and Shaffer [2007].

[34] As long as Π_C^* is strictly larger than Π_N^*, there is a possibility that other common agency equilibria exist (also equilibria that generate different outcomes), but they are all Pareto dominated by the proposed equilibrium.

to a common agent should be set 'low' to induce the retail agent to take into account the positive indirect network effects. On the other hand, according to (19), the wholesale price may sometimes be required to be set 'high' to induce the platforms to set optimal prices on the direct side. If RPM is not feasible, this trade-off between a high and low wholesale price can sometimes lead to excessively high prices on the retail side, and to such an extent that the equilibrium industry profit would be greater in a situation with competing vertical structures. All else being equal, exclusive distribution would reduce double marginalization and induce lower prices on the retail side, due to the competition between the two agents.

Proposition 3 suggests again that there is scope for improving profits by letting the platforms impose RPM clauses, which leads us to our main result.

Proposition 4 *(Common agency – with RPM)*
If RPM is feasible, then a Pareto undominated subgame-perfect common agency equilibrium exists in which each platform adopts an RPM clause $v_C^M = p_r^M$ *and charges a marginal wholesale price* w_C^M *set according to (19) at stage 1, and collects the profit* $\Pi^M/2$ *at stage 2.*

- *If the indirect network effect on side r is negative* $(\partial Q_r^i/\partial q_d^i < 0)$*, then the appropriate RPM clause* v_C^M *is always a maximum (or fixed) price.*

- *If the indirect network effect on side r is positive* $(\partial Q_r^i/\partial q_d^i > 0)$*, then the appropriate RPM clause* v_C^M *is a minimum (or fixed) price iff the diversion ratio between the platforms on side d,* $D_{dd}^{ij} \equiv -\frac{\partial q_d^j}{\partial p_d^i}/\frac{\partial q_d^i}{\partial p_d^i}$*, is sufficiently large,* $D_{dd}^{ij} > \bar{D}(\sigma) \geq 0$*, where* $\bar{D}'(\sigma) < 0$*, and*

$$\sigma \equiv \frac{p_d^M - c_d}{\sum\limits_{s \in \{d,r\}} (p_s^M - c_s)}.$$

Proof. See Appendix A.

The intuition for why RPM works also with competing platforms (in the common agency situation) is the same as in the situation with a monopoly platform. The use of RPM allows the platforms to fix their retail prices at the fully integrated level p_r^M, which in turn allows them to use their marginal wholesale prices (w^i, w^j) to induce prices at the optimal level p_d^M on the direct side of the market. The difference from the monopoly case is that the appropriate wholesale price (19) is different, given that the platforms now are competing on the direct side of the market at the second stage of the game. Thus, the equilibrium RPM clause may turn out to be a minimum price instead of a maximum price. The welfare implications of RPM are therefore also likely to be different.

The intuition for why the appropriate RPM clause may be a minimum price is the following. As the diversion ratio D_{dd}^{ij} between the platforms increases, they tend to set lower prices on the direct side of the market, all else being equal. If the indirect network externalities are positive, the platforms may dampen their incentive to set lower prices on the direct side by charging wholesale prices below cost on the retail side, $w_C^M - c_r < 0$ according to (19).[35] A wholesale price below cost implies that the platform takes a loss on each new customer attracted on the retail side, which in turn, given that network effects are positive, reduces their incentive to set low prices on the direct side. The stronger is the competition (diversion) between the platforms, the lower is the wholesale margin required to mitigate that competition; when the wholesale margin becomes sufficiently low, specifically whenever (19) < (18), the retail agent would prefer to set prices below p_r^M. At that point the RPM clause will have to be a minimum price.

As an example, consider a two-sided market in which (nearly) all the buyers on the direct side of the market are multi-homers. In this case, we should expect the appropriate RPM clause to be a maximum price, according to Proposition 4. To see why, note that a high degree of multi-homing implies that platform j captures fewer new customers when lowering its price p_d^j, because a large share of platform i's customers are already present on platform j; that is, the diversion ratio D_{dd}^{ij} is then very low. In this case, the platforms will tend to set higher prices on the direct side, all else being equal, and thus the appropriate RPM clause is

[35] Note that setting the marginal wholesale price according to (19) could in theory imply that w_C^M becomes negative in some scenarios, if c_r is small enough. We implicitly assume here that (i) c_r is large enough to allow w_C^M to be set according to (19) without it becoming negative, or that (ii) the platforms can require the agent to sell all that it purchases, or require the return of unsold units at the same marginal wholesale price (to prevent the agent from stocking or burning goods at a gain of $-w_C^M > 0$ per unit).

more likely to be a maximum price, according to Proposition 4.

An important determinant for the critical diversion ratio $\bar{D}$ in Proposition 4 turns out to be the relative markups on the two sides of the market. Specifically, Proposition 4 states that $\bar{D}$ becomes smaller when a larger share σ of the total industry markup is earned on the direct side of the market, all else being equal.[36] The intuition is as follows. With positive network effects, the platforms protect their direct side revenues from competition by jointly reducing the marginal wholesale prices offered to the agent. A larger industry markup $p_d^M - c_d$ therefore calls for lower wholesale prices for a given level of platform substitutability, and therefore relaxes the condition ($\bar{D}$ decreases) in Proposition 4 for when the appropriate RPM clause is a minimum price. This suggests that there are important differences between one-sided and two-sided markets, both with regard to the incentives to adopt RPM and the appropriate type of RPM. As an example, consider a one-sided common agency in which the manufacturers engage in (noncontractible) marketing activities that increase demand for their products, after having signed contracts with the retail agent. Importantly, these marketing activities may involve large direct costs, but they generate no direct revenues. Thus, the manufacturers will have to allow themselves strictly positive upstream (wholesale) margins to induce any marketing at all. However, these upstream margins will cause the agent to charge retail prices above the monopoly level, and thus the appropriate RPM clause is always a maximum price in this setting (given of course that marketing is desirable). This is unlike our two-sided market, where the platforms *sometimes* earn substantial revenues (in addition to having some production costs) on the direct side of the market (possibly in addition to the costs of marketing their products). According to our results above, this feature of two-sided markets implies that the appropriate RPM clause sometimes is a minimum price, and not a maximum price, as in the equivalent one-sided setting.

[36] In Appendix A we show that we can write $\bar{D}$ on the form

$$\bar{D}(\sigma) = a + b\frac{1-\sigma}{\sigma}$$

where $a, b > 0$ are functions of all of the "cross-side" diversion ratios as well as the diversion ratio on side r, and σ is the share of the total monopoly markup generated on side d, as defined in Proposition (4) above. Conditions (A20) and (A21) in the appendix provide more details.

Note that the platforms may secure the joint profit Π^M when RPM is feasible, in a similar way to how they may secure the joint profit Π^*_C when RPM is not feasible. Suppose that each platform $i \in \{1, 2\}$ offers to the agent contract terms $\left(v_C^M, w_C^M, F_C^M, f_C^M\right)$, contingent on the agent accepting both offers, and $(v_N^{**}, w_N^{**}, F_N^{**}, f_N^{**})$, contingent on the agent accepting platform i's offer only. Again, because the latter terms are renegotiation proof, we know that they will have to induce the profit $\Pi^{**}_N/2$ for each platform whenever an offer is rejected by the agent. Suppose next that F_C^M and f_C^M are set according to

$$\left(p_r^M - c_d\right) q_d^M + \left(w_C^M - c_r\right) q_r^M + F_C^M = \frac{\Pi^M}{2} \tag{21}$$

and $f_C^M = \Pi^M/2$. These terms will ensure $\Pi^M/2$ to each platform and zero profit to the agent in the second-stage equilibrium, and thus will be accepted if offered by both platforms. Similar to the case without RPM, the terms also ensure that platform i earns a profit of at least $\Pi^M/2$ also when the agent accepts different terms from platform j. Again this will discourage any deviations by platform j. To see this, note that any deviation by platform j away from the terms $\left(v_C^M, w_C^M\right)$, and which are accepted by the agent, will cause the profit of platform i either to increase (in which case the joint profit of the agent and platform j is reduced) or decrease when $q_r^i > 0$. In the latter case, platform i will prefer not to trade with the agent at the final stage, and it will collect the fee $f_C^M = \Pi^M/2$ instead (in which case the joint profit of the agent and platform j is reduced). Thus, again there is no way for a platform to deviate without either (i) hurting its own profit, or (ii) inducing the agent to reject at least one offer. We can therefore conclude that, because $\Pi^M \geq \Pi^{**}_N$, a common agency equilibrium always exists whenever RPM is feasible.

Because the platforms may use minimum RPM to fully dampen competition on the direct side of the market, in a situation where the platforms' customers would benefit from lower prices on both sides, there is a possibility that minimum prices will cause adverse welfare effects, unlike maximum prices. To see this, suppose that network externalities are positive and that (19) < (18). In this case, without a minimum price on the retail side, the platforms will have to balance two concerns. If they set the wholesale prices according to (19), the prices will be too low on the retail side, all else being equal. On the other hand, if they set the wholesale prices according to (18), the prices will be too low on the direct side, all else being equal. In balancing these two concerns, it is therefore likely that the wholesale price w_C^* causes the prices to be too low on both sides of the market simultaneously, as seen from the perspective of the fully integrated monopolist. In this case, a minimum (or fixed) RPM clause will allow the platforms to raise prices on both sides of the market, in a situation where the consumers on each side, given the positive indirect network effects, would benefit from lower prices overall. This intuition is demonstrated by the linear demand example presented in the next section.

We believe that it is relevant to compare our result on minimum prices to Rey and Vergé [2010], who study the effects of RPM in a one-sided market, but with two competing manufacturers who offer nonlinear contracts to two differentiated retail agents (a double common agency). Similar to us, they find that equilibria exist (under certain conditions) where the platforms may use minimum RPM clauses to induce industry-wide monopoly pricing for their products.[37] However, the RPM clauses in Rey and Vergé [2010] serve a somewhat different purpose. In their setting, without RPM, the manufacturers will have to eliminate retail competition by charging marginal wholesale prices in excess of marginal cost (i.e., the contracts are not 'sell-out' contracts). However, because the retailers are then not residual claimants, externalities are introduced at the contracting stage, which in turn implies that the manufacturers charge wholesale prices that are too low to induce monopoly pricing. A pair of minimum or fixed RPM clauses may solve this problem, because they allow the manufacturers to make their retailers into residual claimants, without intensifying the downstream competition at the same time.

In our setting, nonlinear tariffs are sufficiently general to 'solve' the incentive problems at the contracting stage, unlike in Rey and Vergé [2010].[38] Yet, our firms still face incentive

[37] See also Ulsaker [2016], who shows that a pair of 'sales-forcing' contracts can be used instead of minimum RPM in the framework of Rey and Vergé [2010].

[38] In the common agency situation, we show that Pareto dominant equilibria exist in which the platforms are maximizing (15) at the contracting stage, whether or not RPM is used. Rey and Vergé [2010] only look at two-part tariffs, and it is not clear whether more general tariffs would have allowed the firms to fully eliminate the incentive problems at the contracting stage in their setting, even without RPM.

problems at the pricing stage, because the platforms are competing head-to-head on the direct side of the market. In turn, this implies that the prices are distorted away from the monopoly level on both sides of the market simultaneously. When the indirect network externalities are positive, a pair of minimum or fixed RPM clauses may solve this problem, because they allow the platforms to induce the monopoly prices on the retail side of the market, at the same time as they adjust their wholesale prices to eliminate platform competition on the direct side.

Finally, we note that at least one (industry-inefficient) equilibrium outcome exists in addition to the outcome described by Proposition 4. Specifically, a Pareto dominated equilibrium exists in which both platforms use RPM together with a simple two-part tariff (i.e., (7) with either $f^i = 0$ or $f^i = F^i$). In this situation, as described above, the rival platform j will continue to be active (and thus the agent will continue to distribute both products) following a marginal deviation by platform i away from $\left(w_C^M, v_C^M\right)$. Profitable deviations therefore exist for both platforms when simple two-part tariffs are used, and they are unable to sustain the outcome Π^M. An equilibrium thus exists in which the platforms use RPM, but split an industry profit that is smaller than Π^M.[39]

4 A linear demand example

To gain a sense of the potential implications of allowing RPM for the platforms' customers, we consider an example with a single representative customer on each side of the market $s \in \{d, r\}$, each maximizing a surplus function equal to

$$V_s = \sum_{i \in \{1,2\}} q_s^i - \frac{1}{2(1+\varphi_s)} \left(\sum_{i \in 1,2} \left(q_s^i\right)^2 + 2\varphi_s q_s^1 q_s^2 - 2n_s \sum_{i \in 1,2} q_{-s}^i q_s^i \right) - \sum_{i \in \{1,2\}} p_s^i q_s^i. \qquad (22)$$

[39] Unlike Rey and Vergé [2010], however, a continuum of equilibrium outcomes typically does not exist in our setting with two-sided platforms. In Rey and Vergé [2010], a continuum of (weak) equilibria exists because (when fixed RPM is used) each manufacturer's best reply with respect to its own wholesale price is not unique. In our two-sided market, on the other hand, each platform will want to use both the RPM clause v^i and the wholesale price w^i to induce the prices (on both sides) that they prefer at the final stage of the game, instead of using the wholesale price, for example, to redistribute profits (which can be done using the fixed fee F^i).

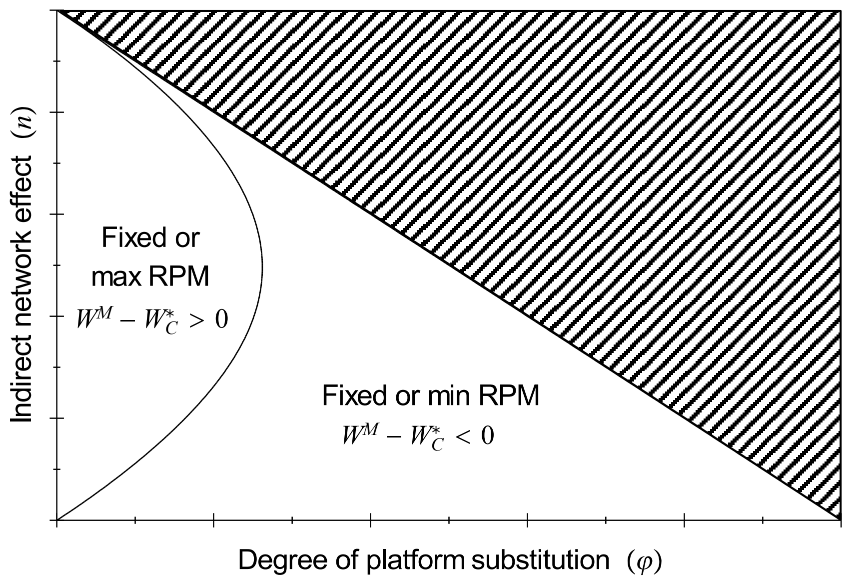

Figure 1: RPM regimes and their welfare effects.

From V_s we obtain the inverse demand functions

$$p_s^i = P_s^i(\mathbf{q}_s, \mathbf{q}_{-s}) = 1 - \frac{1}{1+\varphi_s}\left(q_s^i + \varphi_s q_s^j - n_s q_{-s}^i\right) \tag{23}$$

for $i \in \{1,2\}$ and $s \in \{d,r\}$. In (22) and (23), $n_s \lesseqgtr 0$ is a measure of the indirect network effect and $\varphi_s \in (0,1)$ measures the degree of substitution between the platforms on side s.

When inverting the system (23) for side s, we obtain the following direct demand function as a function of prices on side s and quantities on side $-s$:

$$Q_s^i(\mathbf{p}_s, \mathbf{q}_{-s}) = 1 + n_s \frac{q_{-s}^i - \varphi_s q_{-s}^j}{(1-\varphi_s)(1+\varphi_s)} - \frac{p_s^i - \varphi_s p_s^j}{1-\varphi_s} \tag{24}$$

for $i \in \{1,2\}$ and $s \in \{d,r\}$. To make the analysis analytically tractable, we have to impose some additional symmetry conditions. We therefore consider the case with positive and symmetric network externalities $n_d = n_r = n > 0$, and we assume that the platforms are equally differentiated on both sides of the market, $\varphi_d = \varphi_r = \varphi$. To ensure a unique and economically meaningful solution for the reduced form quantities, our demand parameters must then satisfy $n + \varphi < 1$.

We focus on common agency equilibria in this example. This restriction does not affect our results in any significant way, but makes it easier to derive analytically tractable expressions for the whole parameter space. Equilibria with exclusive agents (see Proposition 3) could be ruled out formally by introducing small economies of scope on the retail side.[40] We also assume that the platforms' marginal production costs are the same on each side of the market, $c_d = c_r = c$.

For the case without RPM, we know that a unique Nash equilibrium $w^i = w^j = w_C^*$ exists at stage 2 of the game, which is the solution to the maximization problem (15). We define $W^M \equiv \sum_{s \in \{d,r\}} V_s^M + \Pi^M$ and $W_C^* \equiv \sum_{s \in \{d,r\}} V_{C,s}^* + \Pi_C^*$ as the overall welfare, with and without RPM, respectively. In Figure 1, we have plotted the loci for which maximum or minimum RPM is appropriate, and for which $W^M - W_C^* > 0$ and $W^M - W_C^* < 0$, respectively.

We can see that when the appropriate RPM clause is a minimum price, then the effect of RPM on overall welfare is negative, whereas when the appropriate RPM clause is a maximum price, the effect is positive. Banning fixed and minimum RPM clauses would therefore be beneficial in our example, whereas banning maximum prices would be detrimental.

Note that this is just an example, but the main intuition should apply more generally. When platform competition is strong, all else being equal, the platforms tend to set prices on the direct side that are too low compared with the prices a monopolist would set. When the wholesale prices are reduced, competition between the platforms is softened (given that the indirect network effects are positive) and the platforms will respond by increasing their prices. A reduction in the wholesale prices, however, will cause the agent to reduce his retail prices as well. This prevents the firms from fully eliminating competition, and as a result the equilibrium prices will end up below the monopoly level on both sides of the market. By imposing a minimum resale price, the platforms can prevent the agent from reducing his retail prices, and hence the prices can be increased to the monopoly level on both sides of the market simultaneously.

[40] Allowing for equilibria with exclusive agents would only increase the welfare gains from allowing platforms to impose maximum prices, for the cases when the degree of substitution between platforms is very weak but positive.

On the other hand, when platform competition is weak, the platforms tend to set prices on the direct side that are too high compared with what a monopolist would do. When the wholesale prices are increased, competition between the platforms is increased (given that the indirect network effects are positive) and the platforms therefore respond by reducing their prices. An increase in the wholesale prices, however, will cause the agent to increase his retail prices. This prevents the platforms from reducing their wholesale prices sufficiently, and as a result the equilibrium prices will end up above the monopoly level on both sides of the market. By imposing a maximum resale price, the platforms can prevent the agent from increasing his retail prices, and hence the prices can be reduced on both sides of the market simultaneously.

5 Conclusion and discussion

This paper has studied the incentives of platforms to adopt RPM in two-sided markets. We have found that two platforms may profitably impose RPM clauses on their agent to internalize the indirect network externalities between the two consumer groups, but also to eliminate competition between the platforms. The appropriate RPM clause –whether a minimum or a maximum price – depends on the signs of the indirect network externalities, the degree of platform competition, and the platforms' relative markups on the two sides of the market. Importantly, our analysis suggests that when (i) network externalities are positive, and (ii) a nontrivial share of the total monopoly markup on each platform is generated on the direct side of the market, adopting minimum RPM clauses may be an effective way for the platforms to induce higher (monopoly) prices on both sides of the market. This suggests that the incentives to use RPM in two-sided markets and the appropriate types of RPM in these markets are different from markets in which all of the industry profit is generated on one side (i.e., one-sided markets). Despite these differences from one-sided markets, our analysis largely validates the prevailing policy regime on RPM in the European Union, which takes a relatively tough stance against minimum and fixed RPM, while allowing a more lenient treatment toward maximum RPM. We believe that these results provide valuable insights for antitrust authorities and regulators. In the following, we briefly discuss the robustness of our results with respect to our assumptions about the contracts and the structure of the market.

We have assumed that the contract signed between a platform and an agent may depend only on the quantity distributed by the agent and (in the case of RPM) on the agent's price. This assumption is not innocuous, although we feel that it is both natural and probably in line with what we observe in reality. As an alternative, we could have assumed that the contracts depend also on the quantities and prices set by the platforms on the direct side of the market. In theory, such contracts could enable the platforms to induce the fully integrated outcome, as an alternative to imposing RPM. In practice, however, it may be costly to use such contracts. First, it may be prohibitively costly for an agent (or for a court) to monitor and verify the prices set on the direct side of the market, especially if these prices are privately negotiated, as they probably sometimes are. The quantities on the direct side, on the other hand, are sometimes directly observed; for example, it is possible to observe the amount of ads in newspapers and commercials on TV. Nevertheless, they would still need to be counted and verified by the agent, which is likely to be costly compared with the cost for the platform of monitoring the agent's retail price (or compared with the cost of simply printing the retail price on the product cover). Finally, we note that in some cases, not even the quantity on the direct side is directly observed by the agent.[41] In sum, we believe that these factors are likely to explain why, for example, newspapers and magazines choose to use fairly simple wholesale contracts in combination with cover pricing (RPM), instead of fixing the prices for ads, or alternatively, giving wholesale discounts based on the amount of ads they sell.

Furthermore, a recurring theme in the vertical contracting literature (in one-sided markets) is that different vertical restraints can be equivalent, in the sense that they allow firms to achieve the same market outcomes. A natural and policy-relevant question is therefore whether the platforms in our two-sided model could sustain the first-best equilibrium by using vertical restraints other than RPM. First, recall that our Lemmas 1 and 2 state that for a nonlinear contract to arise in equilibrium, it cannot have a discontinuity at the equilibrium

[41] As an example, it is not possible to directly observe the number of software developer kits sold by Sony and Microsoft for their video game consoles. This is observed only indirectly, through the amount of software and games released for each system.

quantity because of the cross-group externalities. This means that, for example, market-share contracts or all-units discounts cannot implement the first-best equilibrium in lieu of RPM, as this would require a rebate threshold exactly at the first-best quantity on side r. Second, note that while quantity forcing or rationing tariffs may be sufficient to induce the first-best quantity on side r, they will not enable the platforms to correct their pricing incentives on side d. The reason is that once the first-best quantity has been traded to the retail agent, there is nothing to prevent the platforms from deviating from the first-best price on the direct side. These contracts therefore cannot sustain the first-best equilibrium. Finally, because the first-best solution can only arise when one common agent carries the product of both platforms, it cannot be optimal to assign exclusive territories to agents on side r, or to impose exclusive dealing. So while we of course cannot fully rule out that there may exist some other contract type that allows the platforms to induce the first-best outcome, it seems that RPM is the platforms' optimal choice, at least among the vertical restraints typically considered in the literature.

On the subject of downstream competition, note first that real-life retail markets often comprise imperfectly substitutable agents or outlets, and not isolated retail monopolies such as those in our model. Another layer of complexity would arise if we were to introduce multiple imperfectly substitutable retail locations in our model, as do Rey and Vergé [2010], for example. However, as in Rey and Vergé [2010], we believe that not much would change in our results if we were to simply include a second retail location with its own set of agents. The fact that the second location is imperfectly substitutable for the first matters very little. The offers to the agents should still satisfy their zero-profit condition; it would still be in the platforms' best interest to coordinate and use a common agent at each location (either because there are economies of scale or because the platforms can use RPM); and RPM would make it possible for the platforms to secure the first-best outcome for exactly the same reasons as in our original model. However, because retail locations are competing, equilibrium prices will tend to be lower, which may imply that minimum resale prices will be appropriate over a wider range of parameter values.

Another contentious assumption is that the agents are perfectly substitutable and therefore earn zero profit in equilibrium. The platforms in our model are therefore able to appropriate all of the profits created in equilibrium. Yet, we argue that our results would not change drastically if there were, for example, a single agent with monopoly power downstream. An equilibrium would still exist in which the platforms imposed RPM clauses at the first stage of the game, and the fully integrated prices and profits were maintained at the second stage. The only difference from our original framework would be that each platform might have to leave the agent some rents to keep the rival from inducing the agent to serve it exclusively (assuming the platforms cannot bypass the agent).

Finally, a third possibility is of course a combination of the market conditions listed above. The downstream market may comprise imperfectly competing 'bottlenecks,' as in the one-sided market studied in Rey and Vergé [2010, pp. 945–951]. It is beyond the scope of this paper to describe what would happen in such a market. However, based on the results in Rey and Vergé [2010], in such a market, an equilibrium may not exist in which all agents and platforms are active at the same time.

Appendix A

Proof of Lemma 1 (Monopoly). The proof is structurally identical to the proof of Lemma 2 (see below) and is derived by deleting all derivatives of q_r^j and T^{j*}, as well as all superscripts i, in the proof of Lemma 2. **Q.E.D**.

Proof of Lemma 2 (Duopoly). The proof is comprised of the following three steps.

Step 1. We first show that T^{i*} is continuous at the quantity q_r^* induced by $\mathbf{p}^*$. To see this, assume that T^{i*} is not continuous at the quantity induced by $\mathbf{p}^*$. Then, a marginal deviation, either positive or negative, from $q_r^i(\mathbf{p}^*) = q_r^*$ would cause a discrete change in T^i. If such a deviation causes T^i to 'jump up,' then, since $\partial q_r^i / \partial p_d^i \neq 0$, platform i could adjust p_d^i slightly to change q_r^i, causing a discrete increase in his profits through a larger payment from the agent. If a marginal deviation causes T^i to 'jump down,' then the agent could change p_r^i

slightly (and also, if the agent is a common agent, adjust p_r^j slightly at the same time, to keep q_r^j constant) to induce a discrete reduction in T^i and thus an increase in its profit. In both cases of discontinuity, at least one player (either the platform or the agent) has a profitable deviation. T^{i*} must therefore be continuous at the quantity induced by $\mathbf{p}^*$.

By step 1, T^{i*} has both a right-hand (+) and a left-hand (−) partial derivative with respect to q_r^i at the quantity q_r^*, $\left(\frac{dT^{i*}}{dq_r^i}\right)_+$, and $\left(\frac{dT^{i*}}{dq_r^i}\right)_-$, respectively. For T^{i*} to be differentiable in equilibrium, we require that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ = \left(\frac{dT^{i*}}{dq_r^i}\right)_-$. We show this in two steps, as follows.

Step 2. We first show that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$. For the platforms, two cases for the cross-group externality from side d to side r must be considered.

(i) : With a positive cross-group externality ($\partial q_r^i/\partial p_d^i < 0$), profit maximization requires that

$$\left(\frac{\partial \pi^i}{\partial p_d^i}\right)_- = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(\left(\frac{dT^{i*}}{dq_r^i}\right)_+ - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \geq 0, \tag{A1}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(c_r\frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1} \tag{A2}$$

and

$$\left(\frac{\partial \pi^i}{\partial p_d^i}\right)_+ = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(\left(\frac{dT^{i*}}{dq_r^i}\right)_- - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \leq 0, \tag{A3}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_- \geq \left(c_r\frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1} \tag{A4}$$

for $i \in \{1,2\}$. Hence, we have $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$ according to the platform's first-order profit-maximizing condition.

(ii) : With a negative cross-group externality ($\partial q_r^i/\partial p_d^i > 0$), profit maximization requires that

$$\left(\frac{\partial \pi^i}{\partial p_d^i}\right)_- = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(\left(\frac{dT^{i*}}{dq_r^i}\right)_- - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \geq 0, \tag{A5}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_- \geq \left(c_r \frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1} \tag{A6}$$

and

$$\left(\frac{\partial \pi^i}{\partial p_d^i}\right)_+ = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(\left(\frac{dT^{i*}}{dq_r^i}\right)_+ - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \leq 0, \tag{A7}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(c_r \frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1} \tag{A8}$$

for $i \in \{1, 2\}$. Note that left-hand-side derivative of π^i now includes the left-hand-side derivative of T^{i*} because q_r^i is increasing in p_d^i. Again, it follows from the platform's profit-maximizing condition that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$.

Step 3. We show that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \geq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$. For a common agent, profit maximization requires that

$$\left(\frac{\partial \pi^a}{\partial p_r^i}\right)_- = q_r^i + \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} p_r^k - \frac{\partial q_r^i}{\partial p_r^i}\left(\frac{dT^{i*}}{dq_r^i}\right)_+ - \frac{\partial q_r^j}{\partial p_r^i}\left(\frac{dT^{j*}}{dq_r^j}\right)_- \geq 0, \tag{A9}$$

which, by defining $K_i \equiv \left(q_r^i + \sum_{k\in\{1,2\}}\left(\partial q_r^k/\partial p_r^i\right)p_r^k\right)\left(\partial q_r^i/\partial p_r^i\right)^{-1}$ and rewriting, becomes

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \geq K_i + D_{rr}^{ij}\left(\frac{dT^{j*}}{dq_r^j}\right)_- \tag{A10}$$

(where $D_{rr}^{ij} \equiv -\frac{\partial q_r^j}{\partial p_r^i}/\frac{\partial q_r^i}{\partial p_r^i} \geq 0$ is the diversion ratio to platform j from platform i on side r)

and

$$\left(\frac{\partial \pi^a}{\partial p_r^i}\right)_+ = q_r^i + \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} p_r^k - \frac{\partial q_r^i}{\partial p_r^i}\left(\frac{dT^{i*}}{dq_r^i}\right)_- - \frac{\partial q_r^j}{\partial p_r^i}\left(\frac{dT^{j*}}{dq_r^j}\right)_+ \leq 0, \qquad \text{(A11)}$$

which (in the same way) we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_- \leq K_i + D_{rr}^{ij}\left(\frac{dT^{j*}}{dq_r^j}\right)_+ \qquad \text{(A12)}$$

for $i \neq j \in \{1,2\}$. The left-hand (right-hand)-side derivative of π^a includes the right-hand (left-hand)-side derivative of T^{i*} and the left-hand (right-hand)-side derivative of T^{j*}, because q_r^i is decreasing in p_r^i and q_r^j is increasing in p_r^i. We note that it follows from step 2 that $\left(\frac{dT^{j*}}{dq_r^j}\right)_+ \leq \left(\frac{dT^{j*}}{dq_r^j}\right)_-$. (A10) and (A12) then imply that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \geq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$.

Finally, we note that for an exclusive agent (serving platform i only), we have $K_i \equiv \left(q_r^i + \left(\partial q_r^i/\partial p_r^i\right) p_r^i\right)\left(\partial q_r^i/\partial p_r^i\right)^{-1}$. In addition, the second parts of (A10) and (A12) disappear, because an exclusive agent (for platform i) does not pay any tariff to platform j. Hence, the first-order profit-maximizing conditions for the agent then become $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \geq a_i$ and $\left(\frac{dT^{i*}}{dq_r^i}\right)_- \leq a_i$. Again, it follows from this that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \geq \left(\frac{dT^{j*}}{dq_r^j}\right)_-$.

Together, steps 2 and 3 imply that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ = \left(\frac{dT^{i*}}{dq_r^i}\right)_-$, and thus T^{i*} is differentiable at the quantity q_r^* induced by $\mathbf{p}^*$. **Q.E.D.**

Proof of Proposition 3. Define $\mathbf{p}_C^B(\mathbf{w}) = \left(p_{C,s}^{Bi}(w^i, w^j)\right)$ as the equilibrium prices that simultaneously solve the system of first-order conditions represented by (16) and (17), and let $\mathbf{q}\left(\mathbf{p}_C^B(\mathbf{w})\right) = \left(q_{C,s}^{Bi}(w^i, w^j)\right)$ be the resulting quantities. Define $\Pi\left(\mathbf{p}_C^B(\mathbf{w})\right) = \Pi_C^B(w^i, w^j)$ as the equilibrium overall industry profit in a common agency situation, as a function of the marginal wholesale prices (w^i, w^j) accepted at stage 1. Let w_C^* be the symmetric wholesale price that solves $d\Pi_C^B(w,w)/dw = 0$, that is, the (symmetric) wholesale price that solves the maximization problem (15) for the case without RPM. Define $p_{C,s}^{Bi}(w_C^*, w_C^*) = p_{C,s}^*$ and $q_{C,s}^{Bi}(w_C^*, w_C^*) = q_{C,s}^*$ as the equilibrium price and quantity, respectively, for each firm on side s, when the wholesale prices are equal to w_C^*, and let $\Pi_C^B(w_C^*, w_C^*) = \Pi_C^*$ be the resulting industry profit. Finally, define Π_N^* as the equilibrium industry profit in a version of the game where each platform is assigned an exclusive agent before the game starts (i.e., 'intrinsic exclusive agency'). Consider next a candidate equilibrium, in which the platforms adopt a common agent, and where each platform $i = \{1, 2\}$ offers the agent contract terms (w_C^*, F_C^*, f_C^*), contingent on the agent accepting both offers, and (w_N^*, F_N^*, f_N^*), contingent on the agent accepting platform i's offer only. Given that (w_N^*, F_N^*, f_N^*) are renegotiation proof (as per assumption), we know that these terms will have to induce the profit $\Pi_N^*/2$ for each platform whenever an offer is rejected by the agent. To sustain a common agency equilibrium when $\Pi_C^* \geq \Pi_N^*$, we therefore need that each platform earns at least $\Pi_C^*/2$ when both offers are accepted. Suppose therefore, first that F_C^* is set according to

$$(w_C^* - c_r)\, q_{C,r}^* + \left(p_{C,d}^* - c_d\right) q_{C,d}^* + F_C^* = \frac{\Pi_C^*}{2}, \tag{A13}$$

and second, that $f_C^* = \Pi_C^*/2$. These terms will ensure zero profit to the agent and thus will be accepted if offered by both platforms.

The terms also insulate platform i from any deviations by the rival platform j and the agent, and thus discourages all departures from the proposed equilibrium. To see this, suppose platform j offers terms that include a marginal wholesale price different from w_C^*, and suppose the agent accepts. Two things may happen.

- The deviation causes the second-stage profit of platform i to increase above $\Pi_C^*/2$ whenever $q_r^i > 0$. In this case, by definition (because Π_C^* is the maximum industry profit attainable without RPM), the joint profit of the agent and platform j will have to decrease.

- The second-stage profit of platform i falls below $\Pi_C^*/2$ whenever $q_r^i > 0$. In this case platform i will opt not to be active ($q_r^i = q_d^i = 0$) at the second stage, and collects the fee $f_C^* = \Pi_C^*/2$ instead. Again the joint profit of the agent and platform j will have to decrease.

Thus, there is no way for the agent and a platform to deviate from the proposed equilibrium without hurting their joint profit. What happens after a deviation by platform j is therefore that the agent accepts only one of the contract offers, and in this case we know that each platform earns $\Pi_N^*/2$ in the continuation equilibrium. We may therefore conclude that a common agency equilibrium exists as long as $\Pi_C^* \geq \Pi_N^*$. If the inequality is strict, $\Pi_C^* > \Pi_N^*$, then other common agency equilibria may exist as well, including equilibria that generate different outcomes, but they would all be Pareto dominated by the proposed equilibrium. **Q.E.D.**

Proof of Proposition 4. The first part of the proof follows the same structure as the proof of Proposition 3.

Similar to the proof of Proposition 3, we define as Π_N^{**} the equilibrium industry profit in a version of the game where each platform is assigned an exclusive agent before the game starts (i.e., 'intrinsic exclusive agency'), but assuming they are allowed to use RPM this time (from Proposition 2 we know that RPM is always used). Note also that we have $\Pi^M \geq \Pi_N^{**}$, per definition.

Consider a candidate equilibrium, in which the platforms adopt a common agent, and where each platform $i = \{1, 2\}$ offers the agent contract terms $\left(v_C^M, w_C^M, F_C^M, f_C^M\right)$, contingent on the agent accepting both offers, in which $v_C^M = p_r^M$ is a binding RPM clause and w_C^M is set according to (19), and $(v_N^{**}, w_N^{**}, F_N^{**}, f_N^{**})$, contingent on the agent accepting platform i's offer only. Because the latter terms are renegotiation proof, we know that they will have to induce the profit $\Pi_N^{**}/2$ for each platform whenever an offer is rejected by the agent. Suppose next that F_C^M and f_C^M are set according to

$$\left(p_r^M - c_d\right) q_d^M + \left(w_C^M - c_r\right) q_r^M + F_C^M = \frac{\Pi^M}{2} \tag{A14}$$

and $f_C^M = \Pi^M/2$. Again, these terms will ensure zero profit to the agent and thus will be accepted if offered by both platforms.

Similar to the case without RPM, the terms also insulate platform i from any deviations by the rival platform j and the agent, and thus discourages all departures from the proposed equilibrium. To see this, note that any deviation by platform j away from the terms $\left(v_C^M, w_C^M\right)$, and which are accepted by the agent, may cause one of two outcomes.

- The deviation causes the second-stage profit of platform i to increase above $\Pi^M/2$ whenever $q_r^i > 0$. In this case, by definition (because Π^M is the maximum industry profit), the profit for the agent and platform j will have to decrease.

- The second-stage profit of platform i falls below $\Pi^M/2$ whenever $q_r^i > 0$. In this case platform i will opt not to be active at the second stage ($q_r^i = q_d^i = 0$), and collects the fee $f_C^M = \Pi^M/2$ instead. Again the profit of the agent and platform j will have to decrease (because Π^M is the maximum industry profit).

Thus, again there is no way for the agent and a platform to deviate from the proposed equilibrium without hurting their joint profit. What happens after a deviation by platform j is therefore that the agent accepts only one of the contract offers, and in this case we know that each platform earns $\Pi_N^{**}/2$ in the continuation equilibrium. We may therefore conclude that a common agency equilibrium always exists when RPM is feasible, because we have that $\Pi^M \geq \Pi_N^{**}$. If the inequality is strict, $\Pi^M > \Pi_N^*$, then other common agency equilibria may exist as well, including equilibria that generate different outcomes, but again they are all Pareto dominated by the proposed equilibrium.

Next, we note that, when the marginal wholesale prices are set according to (19), at the fully integrated prices $\mathbf{p}^M$, the condition that the agent would wish to reduce the price p_r^i is that (19) < (18). We have two cases to consider, depending on whether the indirect network effect $\partial Q_r^i/\partial q_d^i$ is negative or positive.

Case 1. Suppose the indirect network effect is negative ($\partial Q_r^i/\partial q_d^i < 0$). The condition that (19) > (18), and therefore that the appropriate resale price is a maximum price, is

$$-\frac{\partial q_r^i}{\partial p_d^i}\sum_{k\in\{1,2\}}\frac{\partial q_r^k}{\partial p_r^i}\left(p_r^M - c_r\right) - \sum_{k\in\{1,2\}}\frac{\partial q_r^k}{\partial p_r^i}\sum_{s\in\{d,r\}}\left(p_s^M - c_s\right)\frac{\partial q_s^j}{\partial p_d^i} - \left(p_d^M - c_d\right)\frac{\partial q_r^i}{\partial p_d^i}\sum_{k\in\{1,2\}}\frac{\partial q_d^k}{\partial p_r^i} > 0. \tag{A15}$$

From the fully integrated monopolist's first-order condition with respect to p_r^i, we have that

$$p_r^M - c_r = \frac{q_r^M + \left(p_d^M - c_d\right) \sum_{k\in\{1,2\}} \frac{\partial q_d^k}{\partial p_r^i}}{-\sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i}}, \tag{A16}$$

which is negative iff $q_r^M < -\left(p_d^M - c_d\right) \sum_{k\in\{1,2\}} \frac{\partial q_d^k}{\partial p_r^i}$. Substituting (A16) into (A15) and rearranging, we obtain the condition

$$q_r^M \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_d^i} + \left(p_d^M - c_d\right) \frac{\partial q_r^j}{\partial p_d^i} \sum_{k\in\{1,2\}} \frac{\partial q_d^k}{\partial p_r^i} - \left(p_d^M - c_d\right) \frac{\partial q_d^j}{\partial p_d^i} \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} > 0. \tag{A17}$$

Given that $\sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_d^i} > 0$ and $\frac{\partial q_r^j}{\partial p_d^i} \leq 0$ (a negative indirect network effect), the left-hand side of (A17) is positive. Hence, the condition always holds and the appropriate resale price is a maximum price.

Case 2. Suppose the indirect network effect is positive ($\partial Q_r^i / \partial q_d^i > 0$). The condition that (19) < (18) and therefore that the appropriate resale price is a minimum price, is the condition (A15). Adding $\left(p_r^M - c_r\right) \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_d^i}$ to both sides of the inequality sign and dividing through by $p_d^M - c_d$, we obtain

$$-\frac{\partial q_d^j}{\partial p_d^i} \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} - \frac{\partial q_r^i}{\partial p_d^i} \sum_{k\in\{1,2\}} \frac{\partial q_d^k}{\partial p_r^i} > \frac{p_r^M - c_r}{p_d^M - c_d} \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_d^i}. \tag{A18}$$

We then make the following definitions: $D_{ss}^{ij} = \partial q_s^j / \partial p_s^i / \left(-\partial q_s^i / \partial p_s^i\right)$ is the (same-side) diversion ratio between the platforms on side $s \in \{d, r\}$, that is, the fraction of the quantity lost by platform i on side s when marginally increasing the price p_s^i, which is captured by the rival platform j on side s; $D_{dr}^{ii} = \partial q_r^i / \partial p_d^i / \left(-\partial q_d^i / \partial p_d^i\right)$ and $D_{rd}^{ii} = \partial q_d^i / \partial p_r^i / \left(-\partial q_r^i / \partial p_r^i\right)$ are the cross-side diversion ratios from side d to r and side r to d, respectively, for platform $i \in \{1, 2\}$, that is, the fraction of the quantity lost by platform i on side s when marginally increasing the price p_s^i, which is captured by the same platform i on the opposite side $-s$;

and $D_{dr}^{ij} = \partial q_r^j / \partial p_d^i / \left(-\partial q_d^i / \partial p_d^i\right)$ and $D_{rd}^{ij} = \partial q_d^j / \partial p_r^i / \left(-\partial q_r^i / \partial p_r^i\right)$ are the cross-side diversion ratios between platforms, from side d to r and side r to d, respectively, that is, the fraction of the quantity lost by platform i on side s when marginally increasing the price p_s^i, which is captured by the rival platform j on the opposite side $-s$. Note that these diversion ratios are more complex than the diversion ratios in a one-sided market. The reason is that the diversion ratio in a two-sided market not only reflects the degree of substitution between the products on one side of the market, but also reflects feedback effects from demand shifting to the other side of the market.

After adding $\frac{\partial q_r^i}{\partial p_d^i} \sum_{k \in \{1,2\}} \frac{\partial q_d^k}{\partial p_r^i}$ to both sides of the inequality sign, and dividing through by $\frac{\partial q_d^i}{\partial p_d^i} \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} > 0$, we may use the definitions for D_{dd}^{ij}, D_{dr}^{ii} and D_{dr}^{ij} to rewrite condition (A18) as

$$D_{dd}^{ij} > -\frac{\sum_{k \in \{1,2\}} \frac{\partial q_d^k}{\partial p_r^i}}{\sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_r^i}} D_{dr}^{ii} - \frac{p_r^M - c_r}{p_d^M - c_d} \left(D_{dr}^{ii} + D_{dr}^{ij}\right). \tag{A19}$$

Finally, using the definitions for D_{rd}^{ii} and D_{rd}^{ij}, we can rewrite condition (A19) as

$$D_{dd}^{ij} > \frac{D_{rd}^{ii} + D_{rd}^{ij}}{1 - D_{rr}^{ij}} D_{dr}^{ii} - \frac{p_r^M - c_r}{p_d^M - c_d} \left(D_{dr}^{ii} + D_{dr}^{ij}\right). \tag{A20}$$

Given that the indirect network effects are positive, we may express this condition as

$$\begin{aligned} D_{dd}^{ij} &> a + b \frac{p_r^M - c_r}{p_d^M - c_d} \\ &= a + b \frac{1-\sigma}{\sigma} \equiv \bar{D}(\sigma), \end{aligned} \tag{A21}$$

where $a, b > 0$ are functions of all the cross-side diversion ratios as well as the diversion ratio on the retail side, and with $\sigma > 0$ representing the share of the total industry markup (positive) that is generated on the direct side. Condition (A20) states that the diversion ratio between the platforms on the direct side must be sufficiently high for a minimum RPM to be appropriate. We can also see that as the markup grows on the direct side (σ increases), the critical diversion ratio $\bar{D}$ shrinks, $\bar{D}'(\sigma) < 0$, and thus it becomes more likely that a minimum (or fixed) price is appropriate. **Q.E.D.**

Appendix B

In the following, we briefly consider the case where $p_r^M - c_r > 0$ and $p_d^M - c_d < 0$. Note that this only has implications for the consideration of whether the appropriate resale price is a maximum or minimum price. There are two cases to consider, depending on whether $\partial Q_r^i / \partial q_d^i < 0$ or $\partial Q_r^i / \partial q_d^i > 0$. We now show that the first case is already covered by Case 1 in the proof of Proposition 4 in Appendix A.

Case 1. Suppose the indirect network effect is negative ($\partial Q_r^i / \partial q_d^i < 0$). We note that the fully integrated monopolist's first-order condition with respect to p_d^i is

$$p_d^M - c_d = \frac{q_d^M + \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_d^i} \left(p_r^M - c_r\right)}{-\sum_{k \in \{1,2\}} \frac{\partial q_d^k}{\partial p_d^i}}. \tag{B1}$$

The right-hand side of (B1) is positive as long as $p_r^M - c_r > 0$, $q_d^M > 0$ and $\sum_{k \in \{1,2\}} \frac{\partial q_d^k}{\partial p_d^i} < 0$. Hence, $p_d^M - c_d > 0$ always, and this case is therefore covered by Case 1 in the proof of Proposition 4 in Appendix A. In practice, we therefore only have one case to consider, which we do next.

Case 2. Suppose the indirect network effect is positive ($\partial Q_r^i / \partial q_d^i > 0$). As before, we find that the condition that (19) < (18), and therefore that the appropriate resale price is a minimum price, is the condition (A15). We note from condition (A21) above that when $p_d^M - c_d > 0$ tends to zero (all else being equal), $\bar{D}$ tends to infinity, which means that the appropriate RPM clause is a maximum price when $p_d^M - c_d$ becomes sufficiently small. This insight extends also to the case where $p_d^M - c_d$ goes from being small but positive to being small but negative. Rewriting the condition, keeping in mind that $p_d^M - c_d < 0$, we find that the inequality (A21) is reversed. Hence, the condition that the appropriate resale price is a minimum price now becomes

$$\begin{aligned} D_{dd}^{ij} &< a + b\frac{p_r^M - c_r}{p_d^M - c_d} \\ &= a + b\frac{1-\sigma}{\sigma} \equiv \bar{D}(\sigma). \end{aligned} \tag{B2}$$

Note that because a and b are positive, as $p_d^M - c_d < 0$ tends to zero, $\bar{D}$ tends to negative infinity, which means that the appropriate RPM clause is again a maximum price when $p_d^M - c_d$ is sufficiently close to zero but negative. However, as $-\left(p_d^M - c_d\right) > 0$ becomes larger, $\bar{D}$ grows larger as well (we note that $\bar{D}$ tends to $a > 0$ as $-\left(p_d^M - c_d\right)$ tends to infinity). Hence, we cannot rule out minimum prices by studying condition (B2) alone. However, we can still show that, also for the case $p_d^M - c_d < 0$, minimum prices require some degree of substitution on at least one side of the market. To see this, consider the case with no substitution on either side of the market, $D_{ss}^{ij} = D_{rd}^{ij} = D_{dr}^{ij} = 0$, for $s \in \{d, r\}$. (We are then essentially back in the monopoly case from Section 2.2.) Condition (B2) is then reduced to

$$0 < D_{rd}^{ii} D_{dr}^{ii} - \frac{p_r^M - c_r}{p_d^M - c_d} D_{dr}^{ii}. \tag{B3}$$

Using the first-order condition with respect to p_d^i for the fully integrated monopolist, we can rewrite condition (B3) as

$$0 < D_{rd}^{ii} D_{dr}^{ii} - \frac{p_r^M - c_r}{\left((p_r^M - c_r) + \dfrac{q_d^M}{\dfrac{\partial q_r^i}{\partial p_d^i}} \right)}, \tag{B4}$$

which never holds given that $p_r^M - c_r > 0$, $q_d^M > 0$, $\partial q_r^i / \partial p_d^i < 0$, and $D_{rd}^{ii} D_{dr}^{ii} < 1$. Hence, for a minimum price ever to be appropriate, we need there to be some degree of competition between the platforms on at least one of the sides.

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