On the Merits of Endogenous Access Pricing

by

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Abstract

Endogenous access pricing (ENAP) is an alternative to the more traditional form of access pricing that sets the access price to reflect the regulator's estimate of the supplier's average cost of providing access. Under ENAP, the access price reflects the supplier's actual average cost of providing access, which varies with realized industry output. We show that, in addition to eliminating the need to estimate industry output accurately and avoiding a divergence between upstream revenues and costs, ENAP can enhance the incentive of a vertically integrated producer to minimize its upstream operating cost.

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1 Introduction

In many settings, a firm that sells an essential input also competes downstream against firms that purchase the input. To illustrate, the owner of a telecommunications network often sells network access to rival retailers of telecommunications services. It is apparent that the established price of the input (the "access price") will affect the outcome of the retail competition between the input supplier and the input buyers in such settings. A high access price can advantage the input supplier by increasing the marginal cost of its retail competitors.

It may be less apparent that the procedure employed to set the access price also can have important implications for industry performance. In particular, endogenous access pricing (ENAP) can offer advantages relative to the more traditional procedure for setting an access price, a procedure that we call exogenous access pricing (EXAP). Under EXAP, before retail competition takes place, a regulator sets a specific access price at which retail rivals can secure access to the network of the incumbent vertically integrated provider (VIP). This access price reflects the regulator's estimate of the VIP's average cost of supplying access. Under ENAP, the regulator explains before retail competition begins how the access price will ultimately be determined, but does not specify a specific, immutable access price. Under ENAP, the unit price that is ultimately charged for access to the incumbent's network is the incumbent's realized average cost of supplying access, i.e., the ratio of the VIP's realized total cost of supplying access to the number of units of access actually supplied.

Fjell et al. (2010) demonstrate that ENAP can help to offset an artificial competitive advantage that EXAP provides to a vertically integrated supplier over its non-integrated retail rivals.[1] To explain this advantage most simply, consider a setting in which: (i) the

[1] Fjell et al. (2010) also explain how ENAP can be implemented in practice. The authors note that the regulator can set an initial access price equal to the expected average cost of supplying access in the coming year. This initial price is the unit price charged for access throughout the year. Then, once the actual cost of supplying access and the amount of access supplied during the year are measured, an additional access surcharge or access rebate is implemented. The surcharge or rebate is calculated to ensure that the final unit price paid for access is the realized average cost of supplying access. Fjell et al. (2010) also describe settings in which access pricing of this type has been implemented in practice.

only industry production cost is the fixed cost of constructing the VIP's network; and (ii) exactly one unit of the VIP's input is required to produce each unit of retail output. The VIP faces no marginal cost of retail production under EXAP in this setting. In contrast, the VIP's non-integrated rivals face a marginal cost equal to the established access price. This cost asymmetry can enable the VIP to serve a relatively large share of the retail market in equilibrium.

ENAP reduces the VIP's incentive to expand its retail output. Increased output by the VIP reduces the access price ultimately charged to retail rivals, and thereby reduces the VIP's wholesale profit. Consequently, the VIP expands its output less aggressively under ENAP than under EXAP, and so its artificial cost advantage is effectively reduced.

Klumpp and Su (2010) identify an additional benefit of ENAP. They demonstrate that ENAP can provide strong incentives for the VIP to undertake network investment that enhances the demand for the retail product supplied by the VIP and its rivals. The VIP realizes that expanded rival output under ENAP obligates them to pay a larger share of the VIP's fixed cost of production in equilibrium, and so increased retail competition can enhance the VIP's incentive to invest in its network under ENAP.

Although they do not analyze ENAP explicitly, Boffa and Panzar (2012) demonstrate the merits of an institutional arrangement that delivers incentives similar to those that arise under ENAP. The authors consider a setting in which retail suppliers jointly own an upstream asset (e.g., a telecommunications network). The fraction of the asset that each retail supplier owns is equal to the supplier's (endogenous) share of equilibrium retail output. This ownership structure provides strong incentives for all suppliers to expand their retail output, in part to reduce the upstream unit cost of production (in light of the prevailing scale economies) and thereby increase upstream profit.

In order to focus on other issues of interest, these pioneering studies of ENAP (and co-ownership of upstream assets) assume that the upstream supplier operates at minimum cost. To develop a complete assessment of the merits of ENAP, it is important to analyze the incentives that ENAP and EXAP provide for cost minimization. The primary purpose of this research is to demonstrate that ENAP often provides stronger incentives for efficient upstream operation than does EXAP.

To understand the rationale for this additional potential benefit of ENAP, recall that the VIP enjoys an artificial retail cost advantage under EXAP. Higher upstream costs enhance this advantage because higher upstream costs increase the prevailing access charge. Under conditions that we identify below, this potential strategic advantage of higher upstream costs can outweigh the direct burden of higher operating costs, and the VIP's profit can increase as its upstream production costs rise.

This potential strategic advantage of higher upstream costs does not arise under ENAP. As noted above, the access price declines as the VIP expands its retail output under ENAP. Consequently, the VIP effectively perceives a marginal cost of expanded retail output under ENAP that it does not perceive under EXAP. As we demonstrate below, ENAP induces all retail rivals to perceive the same marginal cost of retail production regardless of the level of upstream cost, and so increased upstream costs do not increase the VIP's strategic advantage over its retail rivals. Consequently, ENAP often provides stronger incentives than EXAP for upstream cost minimization.

We develop and explain this conclusion more fully as follows. Section 2 describes our formal model. Section 3 analyzes industry performance under EXAP. Section 4 characterizes industry outcomes under ENAP and explains when and why ENAP provides stronger incentives for upstream cost minimization than EXAP. Section 5 identifies additional advantages of ENAP, discusses extensions of our model, and provides concluding observations. The Appendix presents the proofs of all formal conclusions.

2 The Model

We consider a setting in which a vertically integrated provider (VIP) competes against N retail rivals in selling a homogenous product to consumers. The (inverse) consumer demand curve for the homogenous product is $P(Q) = a - b\,Q$, where $a > 0$ and $b > 0$ are constants,

Q is total industry output, and $P(\cdot)$ denotes the market-clearing price.

The VIP is the sole producer of an essential input (e.g., access to the VIP's network). Exactly one unit of the input is required to produce each unit of the retail product. For simplicity, we abstract from retail production costs other than the cost of acquiring the essential input from the VIP.[2] The unit cost of acquiring the input is simply the regulated access price, w, that is charged for the input.

The VIP incurs a fixed cost, F, to produce the input. This fixed cost might be viewed as the cost the VIP incurs to build and maintain its network. The minimum fixed cost required for operation is $\underline{F}$. If the VIP finds it profitable to do so, it can increase F above $\underline{F}$, to a maximum of $\overline{F}$. Such cost inflation serves only to increase the VIP's upstream operating cost – it does not reduce the VIP's downstream cost or improve network performance.[3] Therefore, cost inflation provides no direct value to the VIP. However, as demonstrated below, such cost inflation may benefit the VIP by increasing the access price that is charged to retail rivals.

$\overline{F} - \underline{F}$ can be viewed as the maximum amount of cost inflation the VIP can undertake without detection, and thus without penalty. For analytic simplicity, we assume that additional cost inflation would be detected with sufficiently high probability and penalized sufficiently severely that the VIP never increases F above $\overline{F}$.[4] $\underline{F}$ is assumed to be less than $\frac{a^2}{4b}$ to ensure that industry operation is potentially profitable.[5]

The access price that is charged for the essential input varies with the prevailing access pricing regime. Under exogenous access pricing (EXAP), the access price is $w = \frac{F}{Q^e}$, where Q^e denotes the level of total industry output that the regulator expects to be produced.

[2] We follow Klumpp and Su (2010) in focusing on the tractable setting with linear demand and no variable production costs. The concluding discussion considers alternative settings.

[3] The concluding discussion considers the possibility that cost inflation might provide direct benefits to the VIP.

[4] Alternatively, the VIP might face expected penalty $\Phi(F - \underline{F})$ when it chooses $F \geq \underline{F}$, where $\Phi(\cdot)$ is an increasing, convex function of F. This formulation would provide similar qualitative conclusions, but with additional computational complexity.

[5] It is readily verified that the profit-maximizing retail output for a monopolist is $\frac{a}{2b}$, and the corresponding price is $\frac{a}{2}$. Therefore, the maximum variable profit of the monopolist is $\frac{a^2}{4b}$.

The regulator announces Q^e and F is observed before the industry producers choose their outputs under EXAP. Consequently, the producers consider the identified access price to be fixed and exogenous when they choose their retail outputs.

Under endogenous access pricing (ENAP), the regulator announces that the access price will be $w(Q) = \frac{F}{Q}$, where Q is the level of industry output that ultimately arises. Therefore, under ENAP, each producer realizes that an increase in its retail output will cause the access price that ultimately prevails to decline, *ceteris paribus*.

We will let q_0 denote the VIP's retail output and q_i denote the output of retail rival $i \in \{1, ..., N\}$. The VIP's profit (π_0) is the sum of the revenue it secures from providing access to its retail rivals ($w \sum_{i=1}^{N} q_i$) and its retail profit ($P(Q)\, q_0$, where $Q = \sum_{j=0}^{N} q_j$), less its fixed cost of production (F). Formally:

$$\pi_0(q_0, q_1, ..., q_N, w, F) \;=\; [a - b\,Q]\, q_0 \,+\, w \sum_{i=1}^{N} q_i \,-\, F\,. \tag{1}$$

The corresponding profit of each of the VIP's downstream rivals (π_i) is the product of its retail output (q_i) and its profit margin ($P(Q) - w$). Formally:

$$\pi_i(q_0, q_1, ..., q_N, w) \;=\; q_i\,[a - b\,Q - w] \quad \text{for } i \;=\; 1, 2, ..., N\,. \tag{2}$$

The timing in the model is as follows. First, the regulator announces the access pricing regime that will be implemented. Second, the VIP chooses $F \in \left[\underline{F}, \overline{F}\right]$. Third, the regulator observes F and reports her observation (truthfully). This report determines the prevailing access pricing rule ($w(Q) = \frac{F}{Q}$) if the regulator has implemented ENAP. If she has implemented EXAP, the regulator also announces the industry output she expects to be produced (Q^e), which determines the access price that will prevail ($w = \frac{F}{Q^e}$). Fourth, the VIP and its N retail rivals choose their outputs simultaneously and independently. Finally, the market clearing price is determined, the firms sell their outputs at this price, and the N retail rivals deliver the required access payments to the VIP.

3 Exogenous Access Pricing

We begin our assessment of the relative impacts of EXAP and ENAP on the incentives for upstream cost minimization by examining the outcomes that arise under EXAP. We employ backward induction to characterize these outcomes. Lemma 1 identifies the output that each industry supplier will produce under EXAP, given an established access price. Lemma 2 characterizes $\widehat{w}(F)$, the access price that will prevail under EXAP when the VIP's fixed cost is F. Lemma 3 specifies the VIP's profit under EXAP as a function of F. Finally, Proposition 1 characterizes the VIP's profit-maximizing fixed cost under EXAP.

Lemma 1. *Given access price $\widehat{w}$, the equilibrium output of the VIP under EXAP is $\widehat{q}_0^* = \frac{a + \widehat{w} N}{b [N+2]}$. The equilibrium output of each of the N rivals under EXAP is $\widehat{q}_i^* = \frac{a - 2 \widehat{w}}{b [N+2]}$ for $i = 1, ..., N$.*

Recall that under EXAP, the access price is $\widehat{w} = \frac{F}{Q^e}$. Therefore, to characterize $\widehat{w}$, it is necessary to specify the total output the regulator expects to arise in equilibrium (Q^e). To abstract from forecasts of industry activity that are (intentionally or unintentionally) biased, we assume the regulator estimates the equilibrium output correctly, so $Q^e = \widehat{Q}^*$.[6] Lemma 2 characterizes the access price that will be implemented under EXAP in this case.

Lemma 2. *When the VIP's fixed cost is F, the access price that will be set under EXAP is $\widehat{w}(F) = \frac{1}{2N} \left[a(N+1) - \sqrt{\widehat{G}(F)} \right]$ where $\widehat{G}(F) \equiv a^2 [N+1]^2 - 4 b F N [N+2]$.*

Having identified the access price and the outputs that will arise under EXAP for any given level of fixed cost $F \in [\underline{F}, \overline{F}]$, we can now employ equation (1) to specify the VIP's equilibrium profit under EXAP, given F.

[6] The concluding discussion considers alternative possibilities.

Lemma 3. *For a given fixed cost, F, the VIP's equilibrium profit under EXAP is:*

$$\widehat{\pi}_0^*(F) = \frac{1}{4\,b\,N^2\,[N+2]^2}\left\{2\,a\,N\,[N+4]\sqrt{\widehat{G}(F)} + 4\,b\,F\,N^2\,[N+4]\,[N+2] \right.$$
$$\left. - \; 2\,a^2 N\left[N^2+3\,N+4\right]\right\} - F\,.$$

It can be verified that $\widehat{\pi}_0^{*\prime}(F) \gtreqless 0$ as $F \lesseqgtr \frac{3\,a^2[N-2]}{16\,b\,N}$.[7] Therefore, the VIP's profit-maximizing fixed cost under EXAP, $\widehat{F}^*$, is as specified in Proposition 1.

Proposition 1. *The VIP operates with the cost-minimizing technology under EXAP if it faces fewer than three retail rivals (i.e., $\widehat{F}^* = \underline{F}$ if $N < 3$). In contrast, if the VIP faces three or more rivals and $\underline{F}$ is sufficiently small (e.g., $\underline{F} < \frac{a^2}{16\,b}$), then the VIP will set $\widehat{F}^* = \min\left\{\frac{3\,a^2[N-2]}{16\,b\,N}, \overline{F}\right\} > \underline{F}$ under EXAP.*

The conclusions in Proposition 1 reflect the following considerations. The VIP experiences a gain and a loss when it increases its fixed cost of production above $\underline{F}$. The gain stems from the more pronounced strategic advantage the VIP enjoys in its interaction with retail competitors. The enhanced strategic advantage arises because the access price under EXAP ($\widehat{w} = \frac{F}{Q^e}$) increases as F increases, *ceteris paribus*. Under EXAP, the VIP's rivals incur marginal cost $\widehat{w} > 0$, whereas the VIP's marginal cost of retail output is 0. Therefore, the VIP's marginal cost advantage increases as F, and thus $\widehat{w}$, increases. This increased cost advantage increases the VIP's share of retail output and thus the VIP's profit, *ceteris paribus*.[8]

The loss the VIP incurs when it increases F above $\underline{F}$ is the fraction of the increase in F the VIP is required to bear. Under EXAP, the VIP's expected wholesale profit (i.e., the difference between its revenue from supplying access and the corresponding cost) is:

[7] See the proof of Proposition 1.

[8] Notice from Lemma 1 that the VIP's retail output increases whereas the output of each retail rival declines as $\widehat{w}$ increases under EXAP.

$$\widehat{w} \sum_{i=1}^{N} \widehat{q}_i^{\,*} - F \;=\; \frac{F}{Q^e}\left[Q^e - \widehat{q}_0^{\,*} \right] - F \;=\; - \left[\frac{\widehat{q}_0^{\,*}}{Q^e} \right] F \, . \tag{3}$$

Equation (3) implies that the VIP bears the fraction $\frac{\widehat{q}_0^{\,*}}{Q^e}$ of the fixed cost it implements.

These observations imply that when the VIP faces few retail rivals, it bears a relatively large share of the cost of increasing F while securing an increased retail cost advantage that is of relatively limited value because the VIP faces few rivals. Consequently, as Proposition 1 reports, the VIP refrains from artificial inflation of its fixed cost of production when it faces few (i.e., less than three) retail rivals. In contrast, when the VIP faces many retail rivals, the cost advantage it secures from increasing F is relatively valuable and the fraction of the increase in F it bears is relatively small. Consequently, the VIP may find it profitable to increase F above its minimum feasible level, $\underline{F}$. Indeed, the VIP will undertake such cost inflation unless $\underline{F}$ is so large that even when $F = \underline{F}$, the prevailing access price is sufficiently high that the VIP produces a large share of equilibrium retail output. In this case, an increase in F above $\underline{F}$ obligates the VIP to bear a large fraction of the increase in F while enhancing a strategic cost advantage that is of limited value because rivals are producing relatively little output.

4 Endogenous Access Pricing

To complete our assessment of the relative impacts of EXAP and ENAP on the incentives for upstream cost minimization, we now determine the outcomes that arise under ENAP. Lemma 4 identifies the output that each industry supplier will produce under ENAP, given the VIP's fixed cost, F. Lemma 5 specifies the VIP's profit under ENAP as a function of F. Proposition 2 characterizes the VIP's profit-maximizing choice of F.

Lemma 4. *When the VIP operates with fixed cost F, the equilibrium output of each retail producer under ENAP is* $\widetilde{q}^{\,*} = \frac{a[N+1] + \sqrt{\widetilde{G}(F)}}{2\,b\,[N+1]^2}$, *where* $\widetilde{G}(F) \equiv a^2\,[N+1]^2 - 4\,b\,[N+1]\,F\,N$ *.*

Lemma 4 reports that the VIP produces the same level of output that each of its retail rivals produces under ENAP. This is the case because the VIP, like each retail rival, effectively faces marginal cost $\widetilde{w} = \frac{F}{\widetilde{Q}}$ under ENAP, where $\widetilde{Q}$ denotes total industry output under ENAP. The VIP faces this marginal cost because its wholesale profit under ENAP is:

$$\widetilde{w} \sum_{i=1}^{N} \widetilde{q}_i - F \;=\; \frac{F}{\widetilde{Q}} \left[\widetilde{Q} - \widetilde{q}_0 \right] - F \;=\; - \left[\frac{F}{\widetilde{Q}} \right] \widetilde{q}_0 \;=\; - \, \widetilde{w} \, F \, , \tag{4}$$

where $\widetilde{q}_j$ denotes the output of firm $j \in \{0, 1, ..., N\}$ under ENAP.

Substituting the equilibrium outputs identified in Lemma 4 into equation (1) provides the following expression for $\widetilde{\pi}_0^*(F)$, the VIP's equilibrium profit under ENAP, given fixed cost F.

Lemma 5. $\widetilde{\pi}_0^*(F) = \frac{a^2 [N+1]^2 - \widetilde{G}(F)}{4\, b [N+1]^3} - \frac{F}{N+1}$.

Straightforward differentiation reveals that $\widetilde{\pi}_0^*(\cdot)$ is a strictly decreasing function of F for all $F \in \left[\underline{F} , \overline{F} \right]$. Consequently, the VIP will never artificially inflate its fixed cost of production under ENAP, as Proposition 2 reports.

Proposition 2. *Under ENAP, the VIP always operates with the cost-minimizing technology, i.e.,* $\widetilde{F}^* = \underline{F}$.

Proposition 2 reflects the fact that the VIP effectively faces the same marginal cost ($\widetilde{w}$) that its retail rivals face under ENAP. (Recall equation (4).) Consequently, the VIP does not gain a strategic advantage over its retail rivals when it increases the access price by increasing F above $\underline{F}$. Because an increase in F raises the VIP's operating costs without providing any corresponding strategic advantage, the VIP refrains from any intentional inflation of its operating costs under ENAP.

5 Conclusions

We have shown that endogenous access pricing (ENAP) can provide stronger incentives for upstream cost minimization than exogenous access pricing (EXAP). ENAP enhances the VIP's incentive to reduce its upstream operating cost because it effectively induces the VIP to perceive the same marginal cost of production that its retail rivals face. Consequently, upstream cost increases do not endow the VIP with the same competitive advantage under ENAP that they provide under EXAP.

In principle, a regulator might attempt to limit a firm's incentive to inflate its production cost under EXAP by linking the established access price to an estimate of the firm's minimum feasible operating cost ($\underline{F}$) rather than to the firm's observed cost (F). However, it can be difficult to derive an accurate estimate of $\underline{F}$ in practice.[9] Our findings suggest that ENAP may be an attractive alternative to EXAP quite generally, but particularly when it is difficult to derive precise estimates of the VIP's minimum possible operating cost.

Our formal analysis has considered a simple setting for expositional and analytic convenience. More general results can be derived. For instance, Proposition 2 (which states that the VIP will not intentionally inflate its production costs under ENAP) continues to hold in many settings where the VIP and its rivals operate with positive marginal production costs.[10] Furthermore, although the exact conditions under which the VIP will inflate its fixed cost of production under EXAP are more complex when industry suppliers incur positive marginal production costs, these conditions reflect the basic message of Proposition 1. In particular, the VIP often will set F above $\underline{F}$ when it faces many retail rivals, but will tend to set $F = \underline{F}$ when it faces few rivals.

A VIP may inflate its upstream production cost even under ENAP if such cost inflation offers direct benefits to the VIP. For example, inflated upstream operating costs might take the form of higher wages, benefits, and perquisites for company officials.[11] Even in this case, though, the incentives for cost inflation remain more pronounced under EXAP than under

[9] Kahn et al. (1999) recount the difficulties that regulators encountered in attempting to estimate the minimum possible cost of providing telecommunications services in the United States. Also see Weisman (2002).

[10] This is the case, for example, if the VIP's marginal cost of retail production (c_0) is no less than the marginal cost of the retail rivals (c). If $c_0 < c$, the possibility arises that an increase in the equilibrium access charge caused by an increase in F under ENAP might benefit the VIP by particularly disadvantaging its less efficient retail rivals. Of course, the relatively strong incentive for upstream cost inflation persists under EXAP even when $c_0 < c$.

[11] Blackmon (1994) analyzes such regulatory "abuse."

ENAP, for the reasons identified above.

The VIP refrains from cost inflation in our model under ENAP even though the VIP can increase F above $\underline{F}$ with impunity. This finding implies that the VIP will not raise F above $\underline{F}$ under ENAP if doing so risks a financial penalty. In contrast, the VIP often will continue to increase F above $\underline{F}$ under EXAP when doing so risks financial penalty, provided the expected penalty is not too pronounced.

In closing, we note one additional advantage that ENAP offers relative to EXAP. The access price that is established under EXAP varies with the level of industry output the regulator expects to arise in equilibrium. If the regulator over-estimates (under-estimates) actual industry output, the access price established under EXAP will generate access revenue below (in excess of) the VIP's fixed cost of production (i.e., $\left[\frac{F}{Q^e}\right] \widehat{Q}^* \lesseqgtr F$ as $Q^e \gtreqless \widehat{Q}^*$). This fact has two primary implications. First, the VIP may not secure the intended level of wholesale profit under EXAP, whereas ENAP ensures that wholesale revenue matches wholesale cost. Second, EXAP can invite strategic lobbying to influence the regulator's estimate of equilibrium industry output. Such lobbying serves no purpose under ENAP because the access price that is ultimately established varies only with the realized level of industry output, not with the regulator's estimate of this output.

Appendix

Proof of Lemma 1. Differentiating (1) and (2) provides:

$$\frac{\partial \pi_0}{\partial q_0} \;=\; a - 2\,b\,q_0 - b\sum_{j=1}^{N} q_j \quad \text{and} \quad \frac{\partial \pi_i}{\partial q_i} \;=\; a - b\,q_i - b\,q_0 - b\sum_{j=1}^{N} q_j - w\,. \tag{5}$$

In equilibrium, $\frac{\partial \pi_0}{\partial q_0} = \frac{\partial \pi_i}{\partial q_i} = 0$. Therefore, from (5):

$$a - 2\,b\,q_0 \;=\; b\sum_{j=1}^{N} q_j \;=\; a - b\,q_i - b\,q_0 - w$$

$$\Leftrightarrow \quad b\,q_i \;=\; b\,q_0 - w \quad \Rightarrow \quad b\sum_{i=1}^{N} q_i \;=\; N\,b\,q_0 - w\,N\,. \tag{6}$$

Since $\frac{\partial \pi_0}{\partial q_0} = 0$ in equilibrium, (5) and (6) provide:

$$a - 2\,b\,q_0 - N\,b\,q_0 + w\,N \;=\; 0 \quad \Rightarrow \quad \widehat{q}_0^{\,*} \;=\; \frac{a + w\,N}{b\,[N+2]}\,. \tag{7}$$

(6) and (7) provide:

$$b\,N\,\widehat{q}_i^{\,*} \;=\; N\,b\left[\frac{a + w\,N}{b\,(N+2)}\right] - w\,N \;=\; \frac{a\,N + w\,N^2 - w\,N\,[N+2]}{N+2}$$

$$=\; \frac{a\,N - 2\,w\,N}{N+2} \quad \Rightarrow \quad \widehat{q}_i^{\,*} \;=\; \frac{a - 2\,w}{b\,[N+2]}\,. \quad \blacksquare \tag{8}$$

Proof of Lemma 2. (7) and (8) imply:

$$\widehat{Q}^{\,*} \;=\; q_0^* + \sum_{i=1}^{N} \widehat{q}_i^{\,*} \;=\; \frac{a + w\,N}{b\,[N+2]} + \frac{N\,[a - 2\,w]}{b\,[N+2]} \;=\; \frac{a\,[N+1] - w\,N}{b\,[N+2]}\,. \tag{9}$$

Therefore, when $Q^e \;=\; \widehat{Q}^{\,*}$:

$$w \;=\; \frac{F}{\widehat{Q}^{\,*}} \;=\; \frac{b\,F\,[N+2]}{a\,[N+1] - w\,N} \quad \Rightarrow \quad N\,w^2 - a\,[N+1]\,w + F\,[N+2]\,b \;=\; 0$$

$$\Rightarrow \quad \widehat{w}(F) \;=\; \frac{a\,[N+1] - \sqrt{a^2\,[N+1]^2 - 4\,b\,F\,N\,[N+2]}}{2\,N}. \tag{10}$$

The smaller root here reflects the fact that a smaller w gives rise to larger industry output and welfare. A real solution to (10) exists because:

$$a^2\,[N+1]^2 - 4\,N\,F\,[N+2]\,b \;\geq\; 0 \quad \Leftrightarrow \quad F \;\leq\; \frac{a^2\,[N+1]^2}{4\,b\,N\,[N+2]}\,. \tag{11}$$

Observe that $\frac{[a(N+1)]^2}{4\,b\,N[N+2]} > \frac{a^2}{4\,b}$, since $[N+1]^2 > N\,[N+2]$. ■

Proof of Lemma 3. For expositional ease, we suppress the dependence of $\widehat{w}(\cdot)$ and $\widehat{G}(\cdot)$ on F in the ensuing analysis. From (1), (7), (8), and (9):

$$\widehat{\pi}_0^* = \widehat{q}_0^*\left[a - b\,\widehat{Q}^*\right] + \widehat{w}\sum_{i=1}^{N}\widehat{q}_i^* - F$$

$$= \frac{a+\widehat{w}\,N}{[N+2]\,b}\left[\frac{a+\widehat{w}\,N}{N+2}\right] + \widehat{w}\left[\frac{N\,(a-2\,\widehat{w})}{b\,(N+2)}\right] - F = \frac{H}{b\,[N+2]^2} - F \qquad (12)$$

$$\begin{aligned} \text{where } H &= [a+\widehat{w}\,N]^2 + [N+2]\;\widehat{w}\,N\,[a-2\,\widehat{w}] \\ &= a^2 + N^2\,\widehat{w}^2 + 2\,a\,N\,\widehat{w} + a\,N^2\,\widehat{w} + 2\,a\,\widehat{w}\,N - 2\,N^2\,\widehat{w}^2 - 4\,\widehat{w}^2\,N \\ &= a^2 + a\,N\,\widehat{w}\,[N+4] - \widehat{w}^2\,N\,[N+4] \end{aligned}$$

$$= a^2 + a\,N\,[4+N]\left[\frac{a\,(N+1)-\sqrt{\widehat{G}}}{2\,N}\right] - N\,[N+4]\left[\frac{a^2\,[N+1]^2 + \widehat{G} - 2\,a\,[N+1]\,\sqrt{\widehat{G}}}{4\,N^2}\right]$$

$$= \frac{1}{4\,N^2}\{\,4\,N^2\,a^2 + 2\,a\,N^2\,[N+4]\left[a\,(N+1)-\sqrt{\widehat{G}}\,\right] - N\,[N+4]\left[a^2\,(N+1)^2 + \widehat{G} - 2\,a\,(N+1)\,\sqrt{\widehat{G}}\,\right]\}$$

$$= \frac{1}{4\,N^2}\{\,4\,N^2\,a^2 + 2\,a^2\,N^2\,[N+4]\,[N+1] - \left[2\,a\,N^2\,(N+4)\,\sqrt{\widehat{G}}\,\right] - a^2\,N\,[N+4]\,[N+1]^2 + 2\,a\,N\,[N+4]\,[N+1]\,\sqrt{\widehat{G}} - N\,[N+4]\left[a^2\,(N+1)^2 - 4\,b\,N\,F\,(N+2)\right]\}$$

$$= \frac{1}{4\,N^2}\{\,4\,N^2a^2 + 2\,a^2N^2\,[N+4]\,[N+1] - 2\,a^2N\,[N+4]\,[N+1]^2 - 2\,a\,N^2\,[N+4]\,\sqrt{\widehat{G}} + 2\,a\,N\,[N+4]\,[N+1]\,\sqrt{\widehat{G}} + 4\,b\,F\,N^2\,[N+4]\,[N+2]\,\}$$

$$= \frac{1}{4\,N^2}\{\,-2\,a^2N\left[N^2+3N+4\right] + 2\,a\,N\,[4+N]\,\sqrt{\widehat{G}} + 4\,b\,F\,N^2\,[N+4]\,[N+2]\,\}. \qquad (13)$$

(12) and (13) provide the expression for $\widehat{\pi}_0^*(F)$ specified in the lemma. ■

Proof of Proposition 1. Differentiating $\widehat{\pi}_0^*(F)$ provides:

$$\widehat{\pi}_0^{*\prime}(F) \;=\; \frac{1}{4\,b\,N^2\,[N+2]^2}\left[a\,N\,[4+N]\,\frac{\widehat{G}'(F)}{\sqrt{\widehat{G}}} + 4\,b\,N^2\,(N+4)\,(N+2)\right] - 1$$

$$= \left[\frac{1}{4\,b\,N^2\,[N+2]^2}\right]\left[-\,\frac{4\,a\,N^2\,[N+4]\,[N+2]\,b}{\sqrt{\widehat{G}}} + 4\,b\,N^2\,[N+4]\,[N+2]\right] - 1$$

$$= \frac{4\,N^2\,[N+4]\,[N+2]\,b}{4\,b\,N^2\,[N+2]^2}\left[-\,\frac{a}{\sqrt{\widehat{G}}}+1\right] - 1 \;=\; \frac{N+4}{N+2}\left[-\,\frac{a}{\sqrt{\widehat{G}}}+1\right] - 1\,. \tag{14}$$

(14) implies:

$$\widehat{\pi}_0^{*\prime}(F) \;\gtreqless\; 0 \;\Leftrightarrow\; \left[\frac{N+4}{N+2}\right]\left[-\,\frac{a}{\sqrt{\widehat{G}}}+1\right] \;\gtreqless\; 1$$

$$\Leftrightarrow\; -\,\frac{a}{\sqrt{\widehat{G}}}+1 \;\gtreqless\; \frac{N+2}{N+4} \;\Leftrightarrow\; -\,\frac{a}{\sqrt{\widehat{G}}} \;\gtreqless\; \frac{N+2}{N+4}-1 \;\Leftrightarrow\; -\,\frac{a}{\sqrt{\widehat{G}}} \;\gtreqless\; \frac{-\,2}{N+4}$$

$$\Leftrightarrow\; \frac{a}{\sqrt{\widehat{G}}} \;\lesseqgtr\; \frac{2}{N+4} \;\Leftrightarrow\; \frac{\sqrt{\widehat{G}}}{a} \;\gtreqless\; \frac{N+4}{2} \;\Leftrightarrow\; \sqrt{\widehat{G}} \;\gtreqless\; \frac{[N+4]\,a}{2}$$

$$\Leftrightarrow\; \widehat{G} \;\gtreqless\; \frac{[N+4]^2\,a^2}{4} \;\Leftrightarrow\; [\,a\,(N+1)\,]^2 - 4\,N\,F\,[N+2\,]\,b \;\gtreqless\; \frac{[N+4]^2\,a^2}{4}$$

$$\Leftrightarrow\; a^2\left[(N+1)^2 - \frac{(N+4)^2}{4}\right] \;\gtreqless\; 4\,b\,N\,F\,[N+2\,]$$

$$\Leftrightarrow\; a^2\left[4\,(N+1)^2 - (N+4)^2\right] \;\gtreqless\; 16\,b\,N\,F\,[N+2\,]$$

$$\Leftrightarrow\; a^2\,[\,3\,(N+2)\,(N-2)] \;\gtreqless\; 16\,b\,N\,F\,[N+2\,] \;\Leftrightarrow\; F \;\lesseqgtr\; \frac{3\,a^2\,[N-2\,]}{16\,b\,N}\,. \tag{15}$$

(15) implies that $\frac{\partial \pi_0^*}{\partial F} < 0$ (and so $\widehat{F}^* = \underline{F}$) if $N \leq 2$. In contrast, if $N \geq 3$, then $\widehat{F}^* = \min\left\{\max\left(\underline{F},\, \frac{3\,a^2[N-2]}{16\,b\,N}\right),\, \overline{F}\right\}$. Consequently, $\widehat{F}^* > \underline{F}$ if $\underline{F} < \frac{3\,a^2[N-2]}{16\,b\,N}$. This will be the case if $\underline{F} < \frac{a^2}{16\,b}$, since $r(N) \equiv \frac{N-2}{N}$ is an increasing function of N with $r(3) = \frac{1}{3}$. ■

Proof of Lemma 4. From (1) and (2):

$$\widetilde{\pi}_0(\cdot) \;=\; q_0\,[a - b\,Q] + \frac{F}{q_0 + \sum_{i=1}^N q_i}\sum_{i=1}^N q_i - F\,; \;\text{ and} \tag{16}$$

$$\widetilde{\pi}_i(\cdot) \;=\; q_i\left[a - b\,Q - \frac{F}{q_0 + \sum_{i=1}^N q_i}\right] \quad \text{for } i\; = 1,\; 2, ...,\; N. \tag{17}$$

Differentiating (16) provides:

$$\frac{\partial \pi_0}{\partial q_0} \;=\; a - b\,Q - b\,q_0 - \frac{F\,\sum_{i=1}^{N} q_i}{\left[q_0 + \sum_{i=1}^{N} q_i\right]^2} \;=\; a - b\,Q - b\,q_0 - \frac{F\,[Q - q_0]}{Q^2}\,. \tag{18}$$

Differentiating (17) provides:

$$\begin{aligned}
\frac{\partial \pi_i}{\partial q_i} \;&=\; a - b\,Q - \frac{F}{q_0 + \sum_{i=1}^{N} q_i} - q_i\left[b - \frac{F}{\left[q_0 + \sum_{i=1}^{N} q_i\right]^2}\right] \\
&=\; a - b\,Q - b\,q_i - \frac{F}{q_0 + \sum_{i=1}^{N} q_i}\left[1 - \frac{q_i}{q_0 + \sum_{i=1}^{N} q_i}\right] \\
&=\; a - b\,Q - b\,q_i - \frac{F}{Q}\left[1 - \frac{q_i}{Q}\right] \;=\; a - b\,q_i - b\,Q - \frac{F\,[Q - q_i]}{Q^2}\,.
\end{aligned} \tag{19}$$

Since $\frac{\partial \pi_0}{\partial q_0} = \frac{\partial \pi_i}{\partial q_i} = 0$ in equilibrium, (18) and (19) provide:

$$q_0^* \;=\; q_i^* \;=\; q^* \;\text{ for all } i \;=\; 1, 2, ..., N \quad \Rightarrow \quad Q^* \;=\; q_0^* + \sum_{i=1}^{N} q_i^* \;=\; [N+1]\,q^*\,. \tag{20}$$

Since $\frac{\partial \pi_0}{\partial q_0} = 0$ in equilibrium, (19) and (20) provide:

$$a - b\,[N+1]\,q^* - \frac{F\,N\,q^*}{[N+1]^2\,(q^*)^2} \;=\; 0 \quad \Rightarrow \quad a - b\,[N+1]\,q^* - \frac{F\,N}{q^*\,[N+1]^2} \;=\; 0$$

$$\Rightarrow \quad b\,[N+1]^3\,(q^*)^2 - a\,q^*\,[N+1]^2 + F\,N \;=\; 0. \tag{21}$$

The largest value of q^* that solves (21) is:

$$q^* \;=\; \frac{a\,[N+1]^2 + \sqrt{a^2\,[N+1]^4 - 4\,b\,[N+1]^3\,F\,N}}{2\,b\,[N+1]^3} \;=\; \frac{a\,[N+1] + \sqrt{\widetilde{G}(F)}}{2\,b\,[N+1]^2}\,. \quad \blacksquare \tag{22}$$

Proof of Lemma 5. (16) and (22) provide:

$$\begin{aligned}
\widetilde{\pi}_0^*(F) \;&=\; \widetilde{q}^*\,[a - b(N+1)\widetilde{q}^*] + \left[\frac{F}{(N+1)q^*}\right] N\,\widetilde{q}^* - F \\
&=\; \widetilde{q}^*\,[a - b(N+1)\widetilde{q}^*] + \frac{F\,N}{N+1} - F \;=\; \widetilde{q}^*\,[a - b(N+1)\widetilde{q}^*] - \frac{F}{N+1} \\
&=\; \frac{a\,[N+1] + \sqrt{\widetilde{G}(F)}}{2\,b\,[N+1]^2}\left[a - b(N+1)\frac{a(N+1) + \sqrt{\widetilde{G}(F)}}{2\,b\,(N+1)^2}\right] - \frac{F}{N+1}
\end{aligned}$$

$$= \frac{a^2\,[N+1]^2 - \widetilde{G}(F)}{4\,b\,[N+1]^3} - \frac{F}{N+1}\,. \tag{23}$$

Proof of Proposition 2. Differentiating (23) provides:

$$\widetilde{\pi}_0^{*\prime}(F) = -\left[\frac{\widetilde{G}'(F)}{4\,b\,(N+1)^3}\right] - \frac{1}{N+1} = -\left[\frac{1}{4\,b\,(N+1)^3}\right][-\,4\,b\,(N+1)\,N] - \frac{1}{N+1}$$

$$= \frac{N}{(N+1)^2} - \frac{1}{N+1} = \frac{1}{N+1}\left[\frac{N}{N+1} - 1\right] = -\,\frac{1}{(N+1)^2} < 0. \quad \blacksquare$$

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